Proceedings of "Bi-isotropics'93"

Workshop on novel microwave materials

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Otakaari 5 A, SF-02150 Espoo, Finland

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Abstract
During the first week of February 1993, the Workshop *Bi-isotropics'93* was held at the Helsinki University of Technology in Finland. The topic of the meeting was the electromagnetics and microwave engineering aspects of novel materials, like chiral, nonreciprocal, and even bianisotropic media. This Proceedings contains abstracts, summaries, and other written material relevant to the talks given at the Workshop.

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Introduction

Electromagnetics and microwave technology are progressing these years with considerable pace. One of the frontiers at which major conquests are being made is that of new materials. The interaction of electric and magnetic fields with matter is determined by the physical phenomena taking place within media, but in the electromagnetic description, these effects are reduced to plain material parameters that express the magnitude of the response between the exciting field and the corresponding polarization. It is only during the latest years that microwave community has recognized the potential of novel, more complicated material effects in the design of new components and systems.

Complex materials have, indeed, attracted the attention of several research groups working in the electromagnetics, microwave, millimeter wave, and optical areas. The class of these media — upon which sometimes the label "exotic" is attached — contains several types of materials that loosely can be said to have one thing in common: the ability to exhibit some special effect when excited by the electromagnetic wave. As examples be mentoined chiral, nonreciprocal, nonlinear, gyrotropic, anisotropic, high-T_c superconducting, etc., materials, all of which promise applications in microwave engineering.

Bi-isotropic media are a subgroup of these novel materials. These are isotropic, i.e. they behave similarly regardless the direction of the vector force of the electric or magnetic field. But they are *bi*-isotropic, meaning that there is electrically incited magnetic polarization and vice versa. One might divide bi-isotropic materials in five groups:

- *dielectric* media, consisting microscopically of electrically polarizable entities, which are induced by the electric field
- *magnetic* media, displaying analogously magnetic polarization due to an external magnetic field
- reciprocal *chiral* media, that due to their inherent left- or right-handedness exhibit magnetically caused electric dipole moment density and vice versa; these are also called **Pasteur** media
- *nonreciprocal* media, where the magnetoelectric effect is cophasal unlike in chiral media; also called as **Tellegen** media
- media characterized by any combination of these four effects

In the beginning of February, 1993, Helsinki University of Technology hosted a workshop on novel microwave materials. *Bi-isotropics'93* was the name of the four-day-and-night workshop that attracted 17 participants from six countries: Finland, Russia, Belorussia, France, United Kingdom, and Germany. The focus of the talks was intended to be the theory and applications of bi-isotropic materials in electromagnetics and microwave engineering, but it may be the nature of this rapidly progressing field a reason for the fact that also results on more general, bianisotropic materials were discussed (not to mention the informal interaction sessions where topics like scientific ethics and turmoils in global political structures were reached). Perhaps even *Bianisotropics'93* would be too narrow a title for the next workshop to follow. — Some form of continuation to *Bi-isotropics'93* will certainly materialize; the desire is evident.

This report is a compilation of written material from the presentations held during the workshop. The pages of this "book of abstracts" are diverse: the contributions range from full reports through short summaries to copies of the viewgraphs shown in the oral presentation. Without apologizing for this type of formal shortcomings of my collection, I hope that the information about research results — especially by workshop participants from the former Soviet Union — will percolate on the pages of this report through the electromagnetics and microwave communities of our world.

* * *

Bringing together participants from various countries, and especially from economically unequally-equipped societies, requires commitment in terms of money. I wish to acknowledge the financial assistance from the following three agencies:

- IEEE (The Institute of Electrical and Electronics Engineers) MTT (Microwave Theory and Techniques) Society within Region 8
- URSI (International Union of Radio Science) Finnish National Committee
- The Electromagnetics Laboratory of Helsinki University of Technology

St. Valentine's Day, 1993
Ari Sihvola, Workshop Organizer

Electromagnetics of Novel Microwave Materials
Workshop in Espoo, Finland, 1–4 February 1993

FINAL PROGRAMME

Monday, 1 February

9.00 Ari Sihvola (Helsinki University of Technology, Finland)
Welcome and opening of the workshop
9.30 Ismo Lindell (Helsinki University of Technology, Finland)
Introduction to electromagnetics in novel bi-isotropic media
11.00 Discussion
11.30 Lunch
13.00 Ari Sihvola (Helsinki University of Technology, Finland)
Constitutive relations in bi-isotropic desciption
13.30 Frédéric Mariotte (C.E.S.T.A., France)
and Nader Engheta (University of Pennsylvania, USA)
Theory of guided waves propagation in chiral media:
a step for measuring chiral material parameters
14.30 Discussion
15.30 Arto Hujanen (State Technical Research Centre, Finland)
Measuring electric, magnetic, and chiral material parameters
16.00 Sergei Tretyakov (St. Petersburg State Technical University, Russia)
Comment on bi-slab measurement possibilities
16.30 Discussion

Tuesday, 2 February

9.00 Luk Arnaut (Manchester, United Kingdom)
Microwave chiral research at the UMIST
9.45 Discussion
10.15 Ismo Lindell (Helsinki University of Technology, Finland)
Theory of wave propagation in uniaxial bianisotropic medium
11.15 Discussion
11.30 Ari Viitanen (Helsinki University of Technology, Finland)
Numerical examples of wave propagation in uniaxial bianisotropic medium
12.15 Discussion
12.30 Lunch
14.30 Sergei Tretyakov (St. Petersburg State Technical University, Russia)
Novel uniaxial pseudochiral materials
15.30 Discussion

Wednesday, 3 February

9.00 Sergei Tretyakov (St. Petersburg State Technical University, Russia)
Chiral and bi-isotropic materials in waveguiding applications

9.45 Discussion

10.00 Ari Sihvola (Helsinki University of Technology, Finland)
Macroscopic predictions of materials parameters
for heterogeneous bi media

10.45 Discussion

11.15 Fyodor Fyodorov (Belorussian Academy of Sciences, Minsk)
Covariant methods in the theory of electromagnetic waves

12.30 Discussion

13.00 Lunch

14.00 Visit to the Telecommunications Laboratory
of the State Technical Research Centre in Otaniemi

Thursday, 4 February

10.30 Anatoly Serdyukov (Gomel State University, Belorussia)
Microwave and optical chirality research at the Gomel State University

11.30 Discussion

11.45 Igor Semchenko (Gomel State University, Belorussia)
Particular waves in bi-isotropic media

12.30 Discussion

13.00 Closing of the Workshop

Electromagnetics of Novel Microwave Materials
Workshop in Espoo, Finland, 1–4 February 1993

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Selected bibliography

What follows is a list of recent publications in different journals about the topic of our Workshop: bianisotropic, especially chiral electromagnetics and microwave applications. The list is by no means exhaustive; it only reflects the range and domain of literature followed in the Electromagnetics Laboratory.

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ELECTROMAGNETICS IN NOVEL ISOTROPIC MEDIA

I V Lindell[1]

ABSTRACT

Basic relations of electromagnetic fields in bi-isotropic media are briefly discussed. Such media are more general than the ordinary isotropic dielectric and magnetic media because they can couple electric and magnetic polarizations. Bi-isotropic media have interesting possible applications and the main problem today lies in the fabrication, which has not yet reached an industrial level. However, before starting making such media, their usefulness must be shown theoretically.

INTRODUCTION

Isotropic chiral media have raised interest in the last decade due to their extra medium parameter which gives added freedom in designing microwave and millimeterwave devices. In fact, promising applications in antennas (polarization rotating lenses, compactly packed microstrip antennas), microwave devices (low-loss phase shifters) and radar engineering (reflectionless surfaces) have been suggested, all based on isotropic chiral media.
New applications have induced interest in electromagnetic theory for such media [1, 2]. Actually, chiral media form a subset of the most general isotropic media, the bi-isotropic (BI) media, which again form a subset of the most general linear media, the bianisotropic media. Electromagnetic wave propagation in and reflection from such media is considered in this paper. The basic phenomena, rotation of polarization, in propagation due to chirality and in reflection due to nonreciprocity, of the BI medium, are discussed. Transmission-line analogies are given to treat layered structures of bi-isotropic media. As an example it is shown that a twist polarizer can be simply constructed with just a layer of BI material on a conducting plate.

THE BI MEDIUM

The medium is seen by the electromagnetic fields through the constitutive equations, which for time-harmonic fields ($e^{j\omega t}$ dependence) have the general form involving four medium parameters [3, 4]

$$\mathbf{D} = \epsilon\mathbf{E} + \xi\mathbf{H}, \tag{1}$$

$$\mathbf{B} = \mu\mathbf{H} + \zeta\mathbf{E}, \tag{2}$$

or in terms of other parameters [6, 7, 5], where κ is the chirality parameter and χ, the Tellegen parameter:

$$\xi = (\chi - j\kappa)\sqrt{\mu_o\epsilon_o}, \qquad \zeta = (\chi + j\kappa)\sqrt{\mu_o\epsilon_o}. \tag{3}$$

The relative chiral and Tellegen parameters

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$$\kappa_r = \frac{\kappa}{n}, \qquad \chi_r = \sin\vartheta = \frac{\chi}{n}, \qquad \text{with} \qquad n = \sqrt{\frac{\mu\epsilon}{\mu_o\epsilon_o}} = \sqrt{\mu_r\epsilon_r} \tag{4}$$

will also be applied to simplify the notation. The present form of the constitutive equations is not the only possible one and several other notations are in use in the literature. Different forms of the constitutive relations for BI media are discussed in [5].
It is quite easily shown that, for lossless media, the parameters κ and χ together with ϵ and μ have real values, which with other symbols can be written as $\xi = \zeta^*$ [3]. Also, it can be shown that the BI medium is reciprocal if and only if the Tellegen parameter χ vanishes [10]. Thus, the Tellegen parameter can also be called as the nonreciprocity parameter. Both the chirality parameter κ and the Tellegen parameter χ have a simple physical meaning in terms of plane wave properties, to be discussed shortly.
A medium with nonzero chirality parameter κ requires microscopic constituents without mirror symmetry, e.g., small metallic helices with the same handedness. Actually, first electromagnetic experiments on a man-made chiral medium were done in the 1910's with copper helices randomly embedded in cotton [8]. The Tellegen parameter χ can be theoretically realized by a medium whose microscopic constituents are permanent electric and magnetic dipoles tied together to form similar pairs [9]. It is not known whether this kind of media have yet been actually fabricated.

FIELDS IN BI MEDIUM

Let us consider time-harmonic electric and magnetic current sources $\mathbf{J}$, $\mathbf{J}_m$ giving rise to an electromagnetic field $\mathbf{E}$, $\mathbf{H}$ in a homogeneous BI medium. The Maxwell equations

$$\nabla \times \begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix} = k \begin{pmatrix} \chi_r + j\kappa_r & \eta \\ -1/\eta & -\chi_r + j\kappa_r \end{pmatrix} \begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \mathbf{J} \\ \mathbf{J}_m \end{pmatrix} \tag{5}$$

can be written in uncoupled form for a homogeneous medium by writing the electric and magnetic fields in terms of *wave fields*, also called *self-dual electric fields* [4, 6] $\mathbf{E}_+$, $\mathbf{E}_-$ defined by

$$\mathbf{E}_+ = \frac{1}{\cos\vartheta}(e^{-j\vartheta}\mathbf{E} - j\eta\mathbf{H}), \qquad \mathbf{E}_- = \frac{1}{\cos\vartheta}(e^{j\vartheta}\mathbf{E} + j\eta\mathbf{H}), \tag{6}$$

where $\eta = \sqrt{\mu/\epsilon}$. In fact, (5) can be written as the uncoupled equation pair

$$\nabla \times \mathbf{E}_+ - k_+\mathbf{E}_+ = -j\eta\mathbf{J}_+, \qquad \nabla \times \mathbf{E}_- + k_-\mathbf{E}_- = j\eta\mathbf{J}_-, \tag{7}$$

with

$$k_\pm = k_o(\sqrt{n^2 - \chi^2} \pm \kappa) = k(\cos\vartheta \pm \kappa_r), \tag{8}$$

if the sources $\mathbf{J}$, $\mathbf{J}_m$ are split into two parts $\mathbf{J}_+$ and $\mathbf{J}_-$ as follows:

$$\mathbf{J}_\pm = \frac{1}{2\cos\vartheta}(e^{\pm j\vartheta}\mathbf{J} \pm \frac{1}{j\eta}\mathbf{J}_m). \tag{9}$$

The magnetic field can also be split into two parts, $\mathbf{H} = \mathbf{H}_+ + \mathbf{H}_-$, which have a simple relation to the electric field:

$$\mathbf{H}_\pm = \pm\frac{j}{\eta_\pm}\mathbf{E}_\pm, \qquad \eta_\pm = \eta e^{\mp j\vartheta}, \tag{10}$$

and, similarly, the magnetic source has the decomposition

$$\mathbf{J}_{m\pm} = \frac{1}{2\cos\vartheta}(\pm j\eta\mathbf{J} + e^{\mp j\vartheta}\mathbf{J}_m). \tag{11}$$

With these definitions, the Maxwell equations (7) can in fact be written in the simple isotropic form

$$\nabla \times \mathbf{E}_\pm = -j\omega\mu_\pm\mathbf{H}_\pm - \mathbf{J}_{m\pm}, \tag{12}$$

$$\nabla \times \mathbf{H}_\pm = j\omega\epsilon_\pm\mathbf{E}_\pm + \mathbf{J}_\pm. \tag{13}$$

This means, that if the sources $\mathbf{J}$, $\mathbf{J}_m$ are split into two parts, the $\mathbf{J}_+$ gives rise to "+" fields just like in a simple isotropic medium with the effective medium parameters

$$\epsilon_+ = \epsilon e^{j\vartheta}(\cos\vartheta + \kappa_r), \qquad \mu_+ = \mu e^{-j\vartheta}(\cos\vartheta + \kappa_r), \tag{14}$$

and the minus part to minus fields in another isotropic medium with

$$\epsilon_- = \epsilon e^{-j\vartheta}(\cos\vartheta - \kappa_r), \qquad \mu_- = \mu e^{j\vartheta}(\cos\vartheta - \kappa_r). \tag{15}$$

This idea simplifies the analysis considerably, since at any stage we may find the effect of a BI medium by a cumulative effect of two different isotropic media.

PLANE WAVE IN BI MEDIUM

Plane wave propagation in a BI medium was first analyzed already in 1956 [12]. Let us consider a plane wave propagating in the positive z axis direction in a BI medium. Let the electric field at the plane $z=0$ be written as $\mathbf{E}(0) = \mathbf{E}_+ + \mathbf{E}_-$. Each of the two wave fields $\mathbf{E}_\pm$ see their own isotropic media and propagate with the propagation factors, whence the total field is

$$\mathbf{E}(z) = \mathbf{E}_+ e^{-jk_+z} + \mathbf{E}_- e^{-jk_-z}, \tag{16}$$

$$k_\pm = \omega\sqrt{\mu_\pm\epsilon_\pm} = k(\cos\vartheta \pm \kappa_r). \tag{17}$$

The polarizations of the wave fields are obtained from (7), which outside the sources gives

$$-j\mathbf{u}_z \times \mathbf{E}_\pm(0) \mp k_\pm\mathbf{E}_\pm(0) = 0. \tag{18}$$

It is easy to see that the field vectors are transversal to $\mathbf{u}_z$ and circularly polarized, because they satisfy $\mathbf{E}_\pm \cdot \mathbf{E}_\pm = 0$ and, moreover, the plus wave has the right-hand and, the minus wave, the left-hand polarization. Thus, we may write

$$\mathbf{E}(z) = \frac{1}{2}e^{-jkz\cos\vartheta}\left([\mathbf{E}(0) - j\mathbf{u}_z \times \mathbf{E}(0)]e^{-j\kappa_r kz} + [\mathbf{E}(0) + j\mathbf{u}_z \times \mathbf{E}(0)]e^{j\kappa_r kz}\right)$$

$$= e^{-jkz\cos\vartheta}[\cos(\kappa kz)\overline{\overline{I}}_t - \sin(\kappa kz)\overline{\overline{J}}] \cdot \mathbf{E}(0), \tag{19}$$

$$\overline{\overline{I}}_t = \overline{\overline{I}} - \mathbf{u}_z\mathbf{u}_z, \qquad \overline{\overline{J}} = \mathbf{u}_z \times \overline{\overline{I}}. \tag{20}$$

The dyadic in square brackets is a rotation dyadic which turns the vector $\mathbf{E}(0)$ by an angle $-\kappa_r kz = -\kappa k_o z$ in the right-hand direction when looking in the direction of propagation, $\mathbf{u}_z$. Thus, the chirality parameter κ has the simple physical meaning: it gives the rate of polarization rotation of a propagating linearly polarized plane wave, relative to the rate of phase change of the wave in air: at a distance of λ_o the rotation angle equals $\phi = 2\pi\kappa$. The rotation dyadic can be written in the compact form with a dyadic exponential function

$$\cos(\kappa k_o z)\overline{\overline{I}}_t - \sin(\kappa k_{o\sim})\overline{\overline{J}} = e^{-\kappa k_o z \overline{\overline{J}}}. \tag{21}$$

The polarization rotation in the BI medium is seen to be due to the chirality parameter, because the Tellegen parameter only affects the phase change of the wave. The effect is similar to the Faraday rotation in magnetoplasma or ferrite except that it is the same in all directions of the wave because the BI medium is isotropic. This effect has many possible applications in microwave engineering. For example, a polarization correction for a lens antenna fed by a dipole, by introducing suitably distributed chirality in the lens material, has been suggested [11].

The magnetic field of the plane wave can be written as

$$\mathbf{H}(z) = \mathbf{H}_+(z) + \mathbf{H}_-(z) = \frac{j}{\eta_+}\mathbf{E}_+(z) - \frac{j}{\eta_-}\mathbf{E}_-(z)$$

$$= \frac{1}{2}e^{-jkz\cos\vartheta}\left([\mathbf{E}(0) - j\mathbf{u}_z \times \mathbf{E}(0)]e^{-j\kappa_r kz} + [\mathbf{E}(0) + j\mathbf{u}_z \times \mathbf{E}(0)]e^{j\kappa_r kz}\right) = \frac{1}{\eta}e^{\vartheta+(\pi/2)} \cdot \mathbf{E}(z). \tag{22}$$

This can be interpreted so that the magnetic field of the plane wave is rotated by an angle $\vartheta + \pi/2$ with respect to the electric field. Thus, the Tellegen parameter $\chi = n\sin\vartheta$ is directly related to the excess angle ϑ over the right angle of the electric and magnetic field.

NORMAL REFLECTION FROM AN INTERFACE

Let us consider the plane wave incident from air to an interface of a BI medium at $z = 0$. In normal incidence the problem is easily solved by writing the incident field as a sum of two circularly polarized fields, which do not couple but change places in the reflection. In fact, because the field vector must be rotating in the same direction before ans after reflection, the handedness is changed in reflection, whence the reflected wave sees another isotropic medium than the incident wave.

Let the incident wave arrive from air $z < 0$ to the interface at $z = 0$ of a BI medium in $z > 0$. Writing the incident field at the interface as

$$\mathbf{E}^i = \mathbf{E}^i_+ + \mathbf{E}^i_-, \tag{23}$$

the reflected field is

$$\mathbf{E}^r(0) = R_+\mathbf{E}^i_+ + R_-\mathbf{E}^i_- = \overline{\overline{R}} \cdot \mathbf{E}^i, \tag{24}$$

$$\overline{\overline{R}} = R_+\mathbf{u}_+\mathbf{u}_- + R_-\mathbf{u}_-\mathbf{u}_+, \tag{25}$$

where the circularly polarized unit vectors are

$$\mathbf{u}_+ = \frac{1}{\sqrt{2}}(\mathbf{u}_x - j\mathbf{u}_y), \qquad \mathbf{u}_- = \frac{1}{\sqrt{2}}(\mathbf{u}_x + j\mathbf{u}_y). \tag{26}$$

Inserting

$$R_\pm = \frac{\eta_\pm - \eta_o}{\eta_\pm + \eta_o}, \tag{27}$$

the reflection dyadic (25) can be written as [6]

$$\overline{\overline{R}} = \frac{1}{2}(R_+ + R_-)\overline{\overline{I}}_t - \frac{j}{2}(R_+ - R_-)\overline{\overline{J}} = Re^{\phi\overline{\overline{J}}}, \tag{28}$$

where R denotes the total reflection coefficient and ϕ, the angle of rotation of the reflected polarization,

$$R = \sqrt{\frac{\eta^2 - 2\eta\eta_o\cos\vartheta + \eta_o^2}{\eta^2 + 2\eta\eta_o\cos\vartheta + \eta_o^2}}, \tag{29}$$

$$\tan\phi = -\frac{2\eta\eta_o}{\eta^2 - \eta_o^2}\sin\vartheta = \frac{2\chi}{\epsilon_r - \mu_r}. \tag{30}$$

It is seen that the chirality parameter κ does not affect the reflection at all. On the other hand, the Tellegen parameter χ causes rotation of polarization in reflection. However, $\pm 90^\circ$ rotation is possible only for $\epsilon_r = \mu_r$. The reflection rotation is another quantity giving physical meaning to the Tellegen parameter χ.

The rotation of the reflected field is a demonstration of nonreciprocity of the BI medium with nonzero Tellegen parameter. In fact, a field incident with the polarization of the reflected field does not reflect with the polarization of the incident field but is rotated by another angle ϕ. This effect also exists for BI slabs and it may find important applications in the future when media with nonzero Tellegen parameter can be reliably fabricated.

LAYERED MEDIA, NORMAL INCIDENCE

The previous concepts can be applied to a piecewise homogeneous BI medium where the interfaces are planes perpendicular to the z axis. If a plane wave is incident in $\mathbf{u}_z$ direction to the layered structure, in each homogeneous section the field can be expressed in terms of four waves: one plus and one minus wave in both directions. The plus waves in $+\mathbf{u}_z$ direction are coupled only to minus waves in $-\mathbf{u}_z$ direction and vice versa. So the problem can be split into two scalar problems which can be handled through transmission-line theory except that the properties of the transmission line are different for waves traveling in opposite directions. Thus, we are motivated to study a more general transmission theory [13].

GENERALIZED SCALAR TRANSMISSION-LINE THEORY

The generalized transmission-line equations are written in matrix form as

$$\begin{pmatrix} U(z) \\ I(z) \end{pmatrix}' = -j\omega \begin{pmatrix} a - jb & L \\ C & a + jb \end{pmatrix} \begin{pmatrix} U(z) \\ I(z) \end{pmatrix}, \tag{31}$$

where prime denotes differentiation with respect to z. L is the distributed inductance and C the capacitance on the line. a and b are two new parameters of the transmission line and they may have complex values, although for lossless lines they are real. To emphasize the connection to the field theory, we can write

$$a = \kappa_r\sqrt{LC}, \qquad b = \chi_r\sqrt{LC} = \sin\vartheta\sqrt{LC}. \tag{32}$$

It can be easily shown that the propagation factors for waves propagating in the opposite directions are analogous to those of the previous sections if the plus wave is propagating in the positive z direction

$$\beta^{\pm} = \omega\sqrt{LC - b^2} \pm \omega a = \beta(\cos\vartheta \pm \kappa_r), \qquad \beta = \omega\sqrt{LC}. \tag{33}$$

The corresponding characteristic impedances are

$$Z^{\pm} = \frac{\beta^{\pm} \mp \omega(a + jb)}{\omega C} = \frac{\omega L}{\beta^{\pm} \mp \omega(a - jb)} = Ze^{\mp j\vartheta}, \qquad Z = \sqrt{\frac{L}{C}} = \frac{1}{Y}. \tag{34}$$

The most general voltage function is, then, of the form

$$U(z) = U^{+}e^{-j\beta^{+}z} + U^{-}e^{j\beta^{-}z}, \tag{35}$$

and the corresponding current function,

$$\begin{aligned} I(z) &= I^{+}e^{-j\beta^{+}z} + I^{-}e^{j\beta^{-}z} = Y^{+}U^{+}e^{-j\beta^{+}z} - Y^{-}U^{-}e^{j\beta^{-}z} \\ &= Y[U^{+}e^{-j(\beta^{+}z-\vartheta)} - U^{-}e^{j(\beta^{-}z-\vartheta)}]. \end{aligned} \tag{36}$$

Reflection and transmission at a junction

Let us consider a junction of two transmission lines 1 ($z < 0$) and 2 ($z > 0$) and a voltage wave incident to the junction in line 1, U_1^{+}. There arises a reflected wave and a transmitted wave with the reflection coefficient R and transmission coefficient T defined by

$$U_1^{-} = RU_1^{+}, \qquad U_2^{+} = TU_1^{+}. \tag{37}$$

From continuity of voltage and current at the junction we can solve for the two coefficients:

$$R = \frac{Y_1^{+} - Y_2^{+}}{Y_1^{-} + Y_2^{+}} = \frac{Z_1^{-}(Z_2^{+} - Z_1^{+})}{Z_1^{+}(Z_2^{+} + Z_1^{-})} = \frac{Z_2^{+} - Z_1^{+}}{Z_2^{+} + Z_1^{-}}e^{2j\vartheta_1}, \tag{38}$$

$$T = \frac{Y_1^{-} + Y_1^{+}}{Y_1^{-} + Y_2^{+}} = \frac{Z_2^{+}(Z_1^{+} + Z_1^{-})}{Z_1^{+}(Z_2^{+} + Z_1^{-})} = \frac{2Z_2^{+}\cos\vartheta_1}{Z_2^{+} + Z_1^{-}}e^{j\vartheta_1}. \tag{39}$$

The last expressions are only valid for a lossless line 1 with $Z_1^{\pm} = Z_1e^{\mp j\vartheta_1}$. The coefficients defined for voltage amplitudes appear simpler in terms of admittances. Definitions in terms of current amplitudes are not the same because of $Z^{+} \neq Z^{-}$ and, they appear simpler in terms of impedances. Similar expressions are valid for the reflection from a load admittance Y_L when the admittance Y_2^{+} is replaced by Y_L:

$$R = \frac{Y_1^{+} - Y_L}{Y_1^{-} + Y_L} = \frac{Z_1^{-}(Z_L - Z_1^{+})}{Z_1^{+}(Z_L + Z_1^{-})}. \tag{40}$$

Unlike for ordinary transmission lines, the magnitude of the reflection coefficient is not necessarily unity for the case when all the power is reflected. For example, for open circuit at $z = 0$, we have $Y_L = 0$ and

$$R_{oc} = \frac{Y_1^{+}}{Y_1^{-}} = e^{2j\vartheta_1} \tag{41}$$

instead of $R = 1$. For short circuit $Y_L = \infty$ we have

$$R_{sc} = -1 \tag{42}$$

as for the ordinary line.

Input impedance

The reflection coefficient at a point $z < 0$ in line 1 can be written as

$$R(z) = \frac{U^-(z)}{U^+(z)} = \frac{R(0)U^+e^{j\beta^- z}}{U^+e^{-j\beta^+ z}} = R(0)e^{j(\beta^+ + \beta^-)z} = R(0)e^{j2\beta z \cos\vartheta}. \tag{43}$$

Again, it is noteworthy that the parameter a, or κ_r, does not affect the reflection coefficient. The admittance at the point $z < 0$ of a line loaded by an admittance Y_L is

$$Y_{in}(z) = \frac{I(z)}{U(z)} = \frac{Y^+U^+(z) - Y^-U^-(z)}{U^+(z) + U^-(z)} = \frac{Y^+ - Y^-R(z)}{1 + R(z)}$$
$$= Y\frac{Y_L \cos(\beta d \cos\vartheta + \vartheta) + jY \sin(\beta d \cos\vartheta)}{Y \cos(\beta d \cos\vartheta - \vartheta) + jY_L \sin(\beta d \cos\vartheta)}. \tag{44}$$

The correspondence with the expression of the ordinary line is seen immediately. In fact, setting $\vartheta = 0$, the well-known expression

$$Y_{in} = Y\frac{Y_L \cos\beta d + jY \sin\beta d}{Y \cos\beta d + jY_L \sin\beta d} \tag{45}$$

is obtained.

Impedance matching

The quarter-wavelength matching section of the ordinary transmission line can be generalized for the present transmission line. Requiring that the imaginary part of the input admittance (44) vanishes, gives us the following relation between the parameters βd, ϑ and Y_L/Y:

$$\sin(\beta d \cos\vartheta)[Y_L^2 \cos(\beta d \cos\vartheta + \vartheta) - Y^2 \cos(\beta d \cos\vartheta - \vartheta)] = 0. \tag{46}$$

Excluding the simple solutions $\sin(\beta d \cos\vartheta) = 0$, for which we have $Y_{in} = Y_L$, there is another set of solutions satisfying

$$\tan(\beta d \cos\vartheta) \tan\vartheta = \frac{Y_L^2 - Y^2}{Y_L^2 + Y^2}. \tag{47}$$

Applying this condition in (44) gives us the simple relation

$$Y_{in} = \frac{Y^2}{Y_L}, \tag{48}$$

which is the same as for the ordinary quarter-wavelenth line. Thus, if we wish to match the load admittance Y_L to an admittance Y_o, the line admittance must be chosen as

$$Y = \sqrt{Y_o Y_L}. \tag{49}$$

From (47) it can be seen that, for $\vartheta \neq 0$, the matching length of is changed from the ordinary value of quarter wavelength and shorter matching sections are in fact possible.
In the ultimate case, choosing $\vartheta = \pi/2$ for $Y_L > Y$ and $\vartheta = -\pi/2$ for $Y_L < Y$, (47) becomes

$$\beta d = \left|\frac{Y_L^2 - Y^2}{Y_L^2 + Y^2}\right|, \quad \text{or} \quad d = \frac{\lambda}{4}\frac{2}{\pi}\left|\frac{Y_L^2 - Y^2}{Y_L^2 + Y^2}\right|. \tag{50}$$

This gives always d values smaller than $\lambda/4$. Actually, the length depends on the mismatch of the two impedances: for small mismatch ($Y_L/Y_o \approx 1$) the required line length is small, whereas for the most complete mismatch ($Y_L/Y_o = 0$ or $Y_L/Y_o = \infty$), the length is $\lambda/2\pi \approx \lambda/6$. This means more broadbanded matching than with a quarter-wavelength section of a conventional transmission line.

GENERAL PLANE WAVES IN LAYERED BI MEDIA

Finally, we consider general plane-wave propagation in a piecewise homogeneous layered BI medium with plane-parallel interfaces [14]. In this case, the plus and minus waves couple to one another and the problem cannot be handled in terms of two separate scalar transmission lines as for normal incidence. Also, in the simple isotropic case ($\kappa = 0$, $\chi = 0$) with obliquely incident plane waves, the TE and TM polarizations do not couple to each other, which again leads to two scalar transmission lines. In the general BI problem the TE and TM polarizations have no special importance.

Let us assume that the plane waves are propagating normal to the x axis, which imposes no restriction to generality. For a single plane wave excitation, waves in all layers propagate with the same velocity in the y direction, which implies that all wave vectors have the same component k_y. It turns out that in each layer there are four plane wave components with the same k_y component: two plus waves and two minus waves [15]:

$$\mathbf{E}(\mathbf{r}) = \overrightarrow{\mathbf{E}}_+ \, e^{-j\overrightarrow{\mathbf{k}}_+\cdot\mathbf{r}} + \overrightarrow{\mathbf{E}}_- \, e^{-j\overrightarrow{\mathbf{k}}_-\cdot\mathbf{r}} + \overleftarrow{\mathbf{E}}_+ \, e^{-j\overleftarrow{\mathbf{k}}_+\cdot\mathbf{r}} + \overleftarrow{\mathbf{E}}_- \, e^{-j\overleftarrow{\mathbf{k}}_-\cdot\mathbf{r}}, \tag{51}$$

$$\mathbf{H}(\mathbf{r}) = \overrightarrow{\mathbf{H}}_+ \, e^{-j\overrightarrow{\mathbf{k}}_+\cdot\mathbf{r}} + \overrightarrow{\mathbf{H}}_- \, e^{-j\overrightarrow{\mathbf{k}}_-\cdot\mathbf{r}} + \overleftarrow{\mathbf{H}}_+ \, e^{-j\overleftarrow{\mathbf{k}}_+\cdot\mathbf{r}} + \overleftarrow{\mathbf{H}}_- \, e^{-j\overleftarrow{\mathbf{k}}_-\cdot\mathbf{r}}. \tag{52}$$

The vectors denoted by $\overrightarrow{\mathbf{q}}$ correspond to waves propagating in the positive z direction and, those denoted by $\overleftarrow{\mathbf{q}}$, in the negative z direction. It is assumed that the propagation factors β_+ and β_- defined by

$$\overrightarrow{\mathbf{k}}_\pm = \mathbf{u}_z\beta_\pm + \mathbf{u}_y k_y, \qquad \overleftarrow{\mathbf{k}}_\pm = -\mathbf{u}_z\beta_\pm + \mathbf{u}_y k_y, \tag{53}$$

$$\beta_+ = \sqrt{k_+^2 - k_y^2}, \quad \beta_- = \sqrt{k_-^2 - k_y^2}, \qquad k_\pm = k_o(\sqrt{n^2-\chi^2} \pm \kappa). \tag{54}$$

have a positive real part. Because of the coupling between scalar transmission lines, the general plane-wave problem leads to a concept which can be labeled as vector transmission line.

VECTOR TRANSMISSION-LINE THEORY

In the vector transmission-line (VTL) theory, the transversal field components are represented through the 'vector voltage' $\mathbf{e}$ and 'vector current' $\mathbf{j}$ defined by

$$\mathbf{e} = \overline{\overline{I}}_t \cdot \mathbf{E} \qquad \mathbf{j} = -\mathbf{u}_z \times \mathbf{H} = -\overline{\overline{J}} \cdot \mathbf{H}, \tag{55}$$

The vector voltage in a homogeneous region can be written in terms of those corresponding to the four eigenwaves and the result presented in the compact form with two dyadic propagation factors, one for each direction of propagation,

$$\mathbf{e}(z) = e^{-j\overset{\Rightarrow}{\beta} z}\cdot \overrightarrow{\mathbf{e}}\,(0) + e^{j\overset{\Leftarrow}{\beta} z}\cdot \overleftarrow{\mathbf{e}}\,(0), \tag{56}$$

$$\overset{\Rightarrow}{\beta} = \beta_+ \, \overrightarrow{\mathbf{a}}_+ \overrightarrow{\mathbf{a}}'_+ + \beta_- \, \overrightarrow{\mathbf{a}}_- \overrightarrow{\mathbf{a}}'_-, \qquad \overset{\Leftarrow}{\beta} = \beta_+ \, \overleftarrow{\mathbf{a}}_+ \overleftarrow{\mathbf{a}}'_+ + \beta_- \, \overleftarrow{\mathbf{a}}_- \overleftarrow{\mathbf{a}}'_- \,. \tag{57}$$

The basis vectors $\vec{\mathbf{a}}_{\pm}$, $\overleftarrow{\mathbf{a}}_{\pm}$ are the transversal projections of the circular polarizations of the electric wave fields propagating in the respective $\pm\mathbf{u}_z$ directions:

$$\vec{\mathbf{a}}_{\pm}= \mathbf{u}_x \mp jc_{\pm}\mathbf{u}_y, \qquad \overleftarrow{\mathbf{a}}_{\pm}= \mathbf{u}_x \pm jc_{\pm}\mathbf{u}_y, \qquad c_{\pm} = \cos\theta_{\pm} = \beta_{\pm}/k_{\pm}, \tag{58}$$

and their reciprocal vectors are

$$\vec{\mathbf{a}}'_{\pm}= \mp\frac{1}{J}\mathbf{u}_z\times \vec{\mathbf{a}}_{\mp}, \qquad \overleftarrow{\mathbf{a}}'_{\pm}= \pm\frac{1}{J}\mathbf{u}_z\times \overleftarrow{\mathbf{a}}_{\mp}= \qquad J = \mathbf{u}_z\cdot \vec{\mathbf{a}}_{+} \times \vec{\mathbf{a}}_{-}= j(c_{+}+c_{-}). \tag{59}$$

The basis $\{\vec{\mathbf{a}}_{+}, \vec{\mathbf{a}}_{-}\}$ and the reciprocal basis $\{\vec{\mathbf{a}}'_{+}, \vec{\mathbf{a}}'_{-}\}$ satisfy the orthogonality relations

$$\vec{\mathbf{a}}_{+}\cdot \vec{\mathbf{a}}'_{-}=\vec{\mathbf{a}}_{-}\cdot \vec{\mathbf{a}}'_{+}= 0, \qquad \vec{\mathbf{a}}_{+}\cdot \vec{\mathbf{a}}'_{+}=\vec{\mathbf{a}}_{-}\cdot \vec{\mathbf{a}}'_{-}= 1, \tag{60}$$

and similarly for the other basis vectors $\{\overleftarrow{\mathbf{a}}_{+}, \overleftarrow{\mathbf{a}}_{-}\}$, $\{\overleftarrow{\mathbf{a}}'_{+}, \overleftarrow{\mathbf{a}}'_{-}\}$,. The two propagation dyadics are not independent, in fact, they are related by $\overleftleftarrows{\beta}=\overrightrightarrows{\beta}^{*}$ if $\beta_{\pm}$ and $k_{\pm}$ are real (lossless case). For a lossy medium, the relation is a bit more complicated [15].
The dyadic characteristic admittance for the transmitted wave can be easily identified from expressions of the transmitted vector currents

$$\vec{\mathbf{j}}\,(z) =\overrightrightarrows{Y}\cdot \vec{\mathbf{e}}\,(z), \qquad \overleftarrow{\mathbf{j}}\,(z) = -\overleftleftarrows{Y}\cdot \overleftarrow{\mathbf{e}}\,(z) \tag{61}$$

in the form

$$\overrightrightarrows{Y}= -\mathbf{u}_z\times j\left(\frac{\vec{\mathbf{a}}_{+}\vec{\mathbf{a}}'_{+}}{\eta_{+}} - \frac{\vec{\mathbf{a}}_{-}\vec{\mathbf{a}}'_{-}}{\eta_{-}}\right), \qquad \overleftleftarrows{Y}= \mathbf{u}_z\times j\left(\frac{\overleftarrow{\mathbf{a}}_{+}\overleftarrow{\mathbf{a}}'_{+}}{\eta_{+}} - \frac{\overleftarrow{\mathbf{a}}_{-}\overleftarrow{\mathbf{a}}'_{-}}{\eta_{-}}\right). \tag{62}$$

The relation between the two dyadic admittances can be written for lossless media simply as $\overleftleftarrows{Y}=\overrightrightarrows{Y}^{T*}$, where ^{T} denotes the transpose operation.
For a vector transmission line $z<0$ terminated at $z=0$ with a dyadic admittance $\overrightrightarrows{Y}_L$ defined through the relation

$$\mathbf{j}(0) =\overrightrightarrows{Y}_L\cdot\mathbf{e}(0), \tag{63}$$

a reflection dyadic $\overrightrightarrows{R}\,(z)$ can be defined for waves reflecting into $-\mathbf{u}_z$ direction:

$$\overleftarrow{\mathbf{e}}\,(z) =\overrightrightarrows{R}\,(z)\cdot \vec{\mathbf{e}}\,(z) \tag{64}$$

$\overline{\overline{R}}(z)$ can be expressed in terms of admittance dyadics as follows

$$\overrightrightarrows{R}\,(z) = e^{j\overleftleftarrows{\beta}z}\cdot \overrightrightarrows{R}\,(0)\cdot e^{j\overrightrightarrows{\beta}z}, \qquad \overrightrightarrows{R}\,(0) = (\overleftleftarrows{Y}+\overrightrightarrows{Y}_L)^{-1}\cdot(\overrightrightarrows{Y}-\overrightrightarrows{Y}_L). \tag{65}$$

Finally, an expression for the input admittance $\overrightrightarrows{Y}_{in}\,(z)$ is obtained from the reflection dyadic expression in the form

$$\overrightrightarrows{Y}_{in}\,(z) = [\overrightrightarrows{Y}\cdot e^{-j\overrightrightarrows{\beta}z}\cdot(\overrightrightarrows{Y}-\overrightrightarrows{Y}_L)^{-1} - \overleftleftarrows{Y}\cdot e^{j\overleftleftarrows{\beta}z}\cdot(\overleftleftarrows{Y}+\overrightrightarrows{Y}_L)^{-1}]\cdot$$
$$\cdot[e^{-j\overrightrightarrows{\beta}z}\cdot(\overrightrightarrows{Y}-\overrightrightarrows{Y}_L)^{-1} + e^{j\overleftleftarrows{\beta}z}\cdot(\overleftleftarrows{Y}+\overrightrightarrows{Y}_L)^{-1}]^{-1}. \tag{66}$$

Other forms can be found in [15]. After some algebra, the expression (66) can be seen to reduce to the well-known isotropic formula for normal incidence when $\overline{\overline{Y}}_L$ is a multiple of the unit dyadic $\overline{\overline{I}}_t$.

APPLICATIONS OF VTL THEORY

If we require that the two-dimensional determinant of the reflection dyadic is zero, there arises an equation for the angle of incidence of the plane wave. Since this means that one of the eigenpolarizations (transverse polarizations which do not change in reflection) is zero, the angle can be called the Brewster angle. After considerable algebra, an explicit expression for the Brewster angle θ_B can be obtained in the form

$$\tan^2\theta_B = \pm\frac{4R[R \pm (C_+/C_-)][R \pm (C_-/C_+)]}{C_+C_-(R^2-1)^2}, \tag{67}$$

with

$$C_+ = \sqrt{1-(1/n_+)^2}, \quad C_- = \sqrt{1-(1/n_-)^2}, \tag{68}$$

$$R = \sqrt{\frac{(\eta_+ - \eta_o)(\eta_- - \eta_o)}{(\eta_+ + \eta_o)(\eta_- + \eta_o)}}. \tag{69}$$

For a lossless BI medium we have $\eta_+ = \eta_-^*$, whence R is a real quantity. In this case, (67) has real angle solutions θ_B. For the simple isotropic interface we have, by setting $C_+ = C_- = \sqrt{(\mu_r\epsilon_r - 1)/\mu_r\epsilon_r}$ and $R = (\eta - \eta_o)/(\eta + \eta_o)$, the well-known solutions

$$\tan\theta_{B1} = \sqrt{\frac{\mu_r(\mu_r - \epsilon_r)}{\mu_r\epsilon_r - 1}}, \qquad \tan\theta_{B2} = \sqrt{\frac{\epsilon_r(\epsilon_r - \mu_r)}{\mu_r\epsilon_r - 1}}. \tag{70}$$

There is a new phenomenon, however, because it turns out that two real Brewster angles may exist for certain media [16].

As another example, let us apply the VTL theory to the problem of a BI slab backed by a conducting plane. More applications and tests of the theory can be found in [15]. A perfectly conducting plane at $z = 0$ corresponds to $\overline{\overline{Y}}_L = \infty$, whence the expression for the input admittance (66), for normal incidence, reduces to

$$\overline{\overline{Y}}_{in}(z) = \frac{1}{\eta}[j\cos\vartheta\cot(kz\cos\vartheta)\overline{\overline{I}}_t + \sin\vartheta\overline{\overline{J}}], \tag{71}$$

valid for $z < 0$. The reflection corresponding dyadic has the form

$$\overline{\overline{R}} = (\frac{1}{\eta_o}\overline{\overline{I}}_t + \overline{\overline{Y}}_{in})^{-1}\cdot(\frac{1}{\eta_o}\overline{\overline{I}}_t - \overline{\overline{Y}}_{in}) = R_{co}\overline{\overline{I}}_t + R_{cr}\overline{\overline{J}} \tag{72}$$

with the co and cross-polarized reflection coefficients defined by the expressions

$$R_{co} = -\frac{\eta_o^2\cos^2\vartheta + (\eta^2 - \eta_o^2)\sin^2(kd\cos\vartheta)}{[\eta_o\cos\vartheta\cos(kd\cos\vartheta) + j\eta\sin(kd\cos\vartheta)]^2 - \eta_o^2\sin^2\vartheta\sin^2(kd\cos\vartheta)}, \tag{73}$$

$$R_{cr} = -\frac{2\eta\eta_o\sin\vartheta\sin^2(kd\cos\vartheta)}{[\eta_o\cos\vartheta\cos(kd\cos\vartheta) + j\eta\sin(kd\cos\vartheta)]^2 - \eta_o^2\sin^2\vartheta\sin^2(kd\cos\vartheta)}. \tag{74}$$

For lossless case, these coefficients satisfy $|R_{co}|^2 + |R_{cr}|^2 = 1$.

If we wish to design a twist reflector with the slab of BI medium, the co-polarized reflection is required to vanish. This happens if the following condition between the parameters kd, ϑ and η is valid [17]:

$$\sin(kd\cos\vartheta) = \frac{\eta_o\cos\vartheta}{\sqrt{\eta_o^2-\eta^2}}. \tag{75}$$

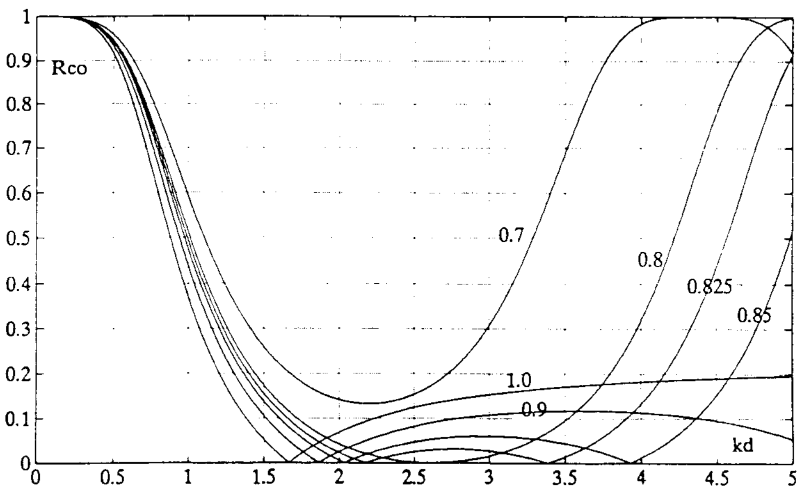
This has real solutions for lossless BI media only for

$$\eta/\eta_o \le |\sin\vartheta| = |\chi_r|, \tag{76}$$

which requires nonzero Tellegen parameter χ.
The twist reflector rotates the reflected field by 90^o and gives a phase shift in reflection defined by

$$R_{cr} = e^{j\psi}, \qquad \psi = \cos^{-1}(\eta/\eta_o\sin\vartheta). \tag{77}$$

This effect can be obtained even for small values of χ if η/η_o is small enough to satisfy the above condition, but, a broad-banded operation is obtained for $\eta/\eta_o = 0.8$ when χ_r is chosen slightly larger than η/η_o, as is seen in the figure.



Co-polarized reflection coefficient for a plane wave incident to a layer of bi-isotropic medium with a conductor backing, as a function of normalized thickness kd. The normalized impedance has the value $\eta/\eta_o = 0.8$ and the normalized Tellegen parameter varies between $\chi_r = 0.7...1.0$. It is seen that the co-polarized reflection is small for a wide band when χ_r has a value slightly larger than η/η_o.

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Dictionary between different definitions of bi-isotropic material parametres

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Due to the fact that there coexist different ways of expressing the constitutive relations of bi-isotropic media, and furthermore, that the quantities used in these may carry the same name without obeying the same definition, it is essential to write explicitely the interconnections between these. The present report serves this purpose[1].

As a practical rule of thumb (often needed in the Workshop's discussions), which will be derived in this report, among others, is the following:

- IF YOU SEE SOMEBODY PRESENTING RESULTS OF CHIRAL MEDIA, AND SHE (HE) TALKS ABOUT CHIRALITY ADMITTANCES (ξ_c), AND YOU WISH TO KNOW THE DIMENSIONLESS CHIRALITY VALUE OF THIS PIECE OF MATERIAL, MULTIPLE THE ADMITTANCE FIGURE BY THE FREE SPACE IMPEDANCE, $\eta_0 \sim 377\,\Omega$, AND YOU WILL HAVE THE CHIRALITY PARAMETER κ. — This is approximate, and exact only for materials where $\mu = \mu_0$, but the experimental results so far show permeabilities not much different from μ_0. Should you desire more accurate transformation formulae, please keep on reading :-s

1 The basic relations

Most of the presentations given in the present *Bi-isotropics'93* workshop speak about bi-isotropic media using the language and terms of the following constitutive rela-

[1] Much of the material to follow is common to that in [1].

tions:

$$\bar{D} = \epsilon \bar{E} + (\chi - j\kappa)\sqrt{\mu_0 \epsilon_0}\bar{H} \tag{1}$$

$$\bar{B} = \mu \bar{H} + (\chi + j\kappa)\sqrt{\mu_0 \epsilon_0}\bar{E} \tag{2}$$

The relations give the effect of the electric ($\bar{E}$) and magnetic ($\bar{H}$) field strengths on the electric ($\bar{D}$) and magnetic ($\bar{B}$) flux densities. The effects can be seen to be isotropic (*i.e.* the direction of the fields does not matter), because the coefficients representing the linear relation between these are scalars. These parameters are, in addition to the normally encountered permittivity ϵ and permeability μ of the material, those that describe the magnetoelectric coupling. The degree of chirality is contained in κ, a dimensionless (Pasteur) parameter, and χ is the (also dimensionless) Tellegen parameter measuring the nonreciprocity of this general biisotropic material. ϵ_0 and μ_0 are the permittivity and permeability of the vacuum. For $\chi = 0$, the medium is (nonreciprocal) chiral, and it can also be termed Pasteur medium. The case when the chirality vanishes, $\kappa = 0$, represents a nonreciprocal medium, that can be called Tellegen medium, because the form the constitutive relations that Tellegen first proposed for a nonreciprocal medium was of the form of (1), (2) with $\kappa = 0$.

It is worth noting that these relations implicitely assume sinusoidal time dependence (followed with the convention $\exp(j\omega t)$ in this paper), meaning that the parameters are dispersive. The time dependence comes through the inverse Fourier transform, and in order to render real electromagnetic fields (to represent a physical quantity), the parameters have to be real functions of $j\omega$. On the other hand, the four material parameters of a lossless medium have to be real [2], leading to the conclusion that ϵ, μ, χ have to be even functions of ω, and κ an odd function. Hence, there is no chirality in electro- or magnetostatics.

In the research of chiral media, there are many different constitutive relations in use (see, for example [3]) due to the possibilities in connecting the electric and magnetic quantities with another. In the following, the parameters in different sets of constitutive relations are related with each other. This will help translate electromagnetic results derived in one system to the other system. The most common chirality representations will be treated and their extensions to the nonreciprocal chiral regime, i.e., the chiral constitutive relations that are considered will be generalized for bi-isotropic media.

The problem will be that in different systems, the same name has been given to parameters that are not absolutely the same, for example permittivity and permeability. Therefore bookkeeping is essential which system's permittivity is considered as one speaks about permittivity. In the following, all relations are referred against the constitutive relations (1), (2) and this permittivity is plain ϵ and permeability plain μ in the following analysis. The corresponding parameters in other systems are denoted by subindices.

2 Post relations

The Post [4] set of constitutive relations for chiral media, also derived phenomenologically by Jaggard, Mickelson, and Papas [5], is the following

$$\bar{D} = \epsilon_{\mathrm{PJ}}\bar{E} - j\xi_c\bar{B} \tag{3}$$

$$\bar{H} = \frac{1}{\mu_{\mathrm{PJ}}}\bar{B} - j\xi_c\bar{E} \tag{4}$$

where the subindices in ϵ and μ now refer to the system, and the chirality comes forth through the chirality admittance ξ_c. To complete the relations in order to cover general bi-isotropic media, a fourth scalar parameter describing the nonreciprocity is needed. In harmony with the chirality admittance ξ_c, a nonreciprocity susceptance ψ_n is here suggested, which also has the dimension of amperes/volt. The relations, after the inclusion of nonreciprocity, look after that like

$$\bar{D} = \epsilon_{\mathrm{PJ}}\bar{E} + \psi_n\bar{B} - j\xi_c\bar{B} \tag{5}$$

$$\bar{H} = \frac{1}{\mu_{\mathrm{PJ}}}\bar{B} - \psi_n\bar{E} - j\xi_c\bar{E} \tag{6}$$

The connections between the parameters in (5), (6) and (1), (2) can be worked out and are the following

$$\epsilon_{\mathrm{PJ}} = \epsilon - \frac{\mu_0\epsilon_0}{\mu}(\chi^2 + \kappa^2) \tag{7}$$

$$\mu_{\mathrm{PJ}} = \mu \tag{8}$$

$$\psi_n = \frac{\sqrt{\mu_0\epsilon_0}}{\mu}\chi \tag{9}$$

$$\xi_c = \frac{\sqrt{\mu_0\epsilon_0}}{\mu}\kappa \tag{10}$$

and, the other way

$$\epsilon = \epsilon_{\mathrm{PJ}} + \mu_{\mathrm{PJ}}(\psi_n^2 + \xi_c^2) \tag{11}$$

$$\mu = \mu_{PJ} \tag{12}$$

$$\chi = \mu_{PJ}\psi_n/\sqrt{\mu_0\epsilon_0} \tag{13}$$

$$\kappa = \mu_{PJ}\xi_c/\sqrt{\mu_0\epsilon_0} \tag{14}$$

3 Condon-Tellegen relations

The constitutive relations of chiral media that have received their label after Condon [6], look like the following, permitting an explicit time dependence in the fields:

$$\bar{D} = \epsilon_C\bar{E} - \chi_C\frac{\partial\bar{H}}{\partial t} \tag{15}$$

$$\bar{B} = \mu_C\bar{H} + \chi_C\frac{\partial\bar{E}}{\partial t} \tag{16}$$

with the parameter χ_C (now <u>not</u> a nonreciprocity parameter) measuring the material's magnitude of chirality (dimension sec^2/meters). This is a reciprocal chiral material. On the other hand, a material that is not chiral but is nonreciprocal was suggested by Tellegen [7] to obey the following constitutive relations

$$\bar{D} = \epsilon_T\bar{E} + \gamma_T\bar{H} \tag{17}$$

$$\bar{B} = \mu_T\bar{H} + \gamma_T\bar{E} \tag{18}$$

where γ_T, dimensionally sec/meters, measures the nonreciprocity of the material. These material relations can be combined for a set of bi-isotropic constitutive relations:

$$\bar{D} = \epsilon_{CT}\bar{E} + \gamma_{CT}\bar{H} - \chi_{CT}\frac{\partial\bar{H}}{\partial t} \tag{19}$$

$$\bar{B} = \mu_{CT}\bar{H} + \gamma_{CT}\bar{E} + \chi_{CT}\frac{\partial\bar{E}}{\partial t} \tag{20}$$

For time-harmonic field dependence, the connection of these material parameters to those in (1), (2) is

$$\epsilon_{CT} = \epsilon \tag{21}$$

$$\mu_{\mathrm{CT}} = \mu \tag{22}$$

$$\gamma_{\mathrm{CT}} = \chi\sqrt{\mu_0\epsilon_0} \tag{23}$$

$$\chi_{\mathrm{CT}} = \kappa\sqrt{\mu_0\epsilon_0}/\omega \tag{24}$$

4 Drude-Born-Fedorov relations

The relations emphasizing the nonlocal character of a chiral medium in its interaction with the electromagnetic field,

$$\bar{D} = \epsilon_{\mathrm{DBF}}(\bar{E} + \beta\nabla \times \bar{E}) \tag{25}$$

$$\bar{B} = \mu_{\mathrm{DBF}}(\bar{H} + \beta\nabla \times \bar{H}) \tag{26}$$

have been in much use by Lakhtakia, Varadan, and Varadan, and termed by them [8] after Drude, Born, and Fedorov. (An advantage in these relations is that these are not restricted to Fourier space, and the fact that from these, it can be directly seen that chirality vanishes for electro- and magnetostatics.) Here the chirality parameter β has the dimension of length. One way of incorporating nonreciprocity in these relations would be through the Tellegen parameter:

$$\bar{D} = \epsilon_{\mathrm{DBF}}(\bar{E} + \beta\nabla \times \bar{E}) + \gamma_{\mathrm{T}}\bar{H} \tag{27}$$

$$\bar{B} = \mu_{\mathrm{DBF}}(\bar{H} + \beta\nabla \times \bar{H}) + \gamma_{\mathrm{T}}\bar{E} \tag{28}$$

However, as often only time-harmonic field dependencies are considered in electromagnetic applications, it might be justified to tolerate complex quantities and the appearance of the imaginary unit j in these relations (like happens in relations (1), (2) and (5), (6)). This suggests completing the bi-isotropic DBF relations in the form

$$\bar{D} = \epsilon_{\mathrm{DBF}}[\bar{E} + (\beta + j\alpha)\nabla \times \bar{E}] \tag{29}$$

$$\bar{B} = \mu_{\mathrm{DBF}}[\bar{H} + (\beta - j\alpha)\nabla \times \bar{H}] \tag{30}$$

where now the nonreciprocity parameter α has the same dimension of length as the chirality parameter β.

The conversion of the material parameters between the sets (1), (2) and (29), (30) is, for time-harmonic fields

$$\epsilon_{\text{DBF}} = \epsilon\left[1 - (\chi^2 + \kappa^2)\frac{\mu_0\epsilon_0}{\mu\epsilon}\right] \tag{31}$$

$$\mu_{\text{DBF}} = \mu\left[1 - (\chi^2 + \kappa^2)\frac{\mu_0\epsilon_0}{\mu\epsilon}\right] \tag{32}$$

$$\alpha = \frac{\chi\sqrt{\mu_0\epsilon_0}/\omega}{\mu\epsilon - (\chi^2 + \kappa^2)\mu_0\epsilon_0} \tag{33}$$

$$\beta = \frac{\kappa\sqrt{\mu_0\epsilon_0}/\omega}{\mu\epsilon - (\chi^2 + \kappa^2)\mu_0\epsilon_0} \tag{34}$$

and, inversely,

$$\epsilon = \frac{\epsilon_{\text{DBF}}}{1 - k_{\text{DBF}}^2(\alpha^2 + \beta^2)} \tag{35}$$

$$\mu = \frac{\mu_{\text{DBF}}}{1 - k_{\text{DBF}}^2(\alpha^2 + \beta^2)} \tag{36}$$

$$\chi = \frac{\omega\mu_{\text{DBF}}\epsilon_{\text{DBF}}\alpha/\sqrt{\mu_0\epsilon_0}}{1 - k_{\text{DBF}}^2(\alpha^2 + \beta^2)} \tag{37}$$

$$\kappa = \frac{\omega\mu_{\text{DBF}}\epsilon_{\text{DBF}}\beta/\sqrt{\mu_0\epsilon_0}}{1 - k_{\text{DBF}}^2(\alpha^2 + \beta^2)} \tag{38}$$

with

$$k_{\text{DBF}}^2 = \omega^2\mu_{\text{DBF}}\epsilon_{\text{DBF}} \tag{39}$$

5 Discussion

A general bi-isotropic medium requires four scalar material parameters in its characterization. In this presentation, instead of connecting the electromagnetic quantities in a classical way, through a matrix [9]

$$\begin{pmatrix} \bar{D} \\ \bar{B} \end{pmatrix} = \begin{pmatrix} \epsilon & \gamma \\ \beta & \mu \end{pmatrix} \begin{pmatrix} \bar{E} \\ \bar{H} \end{pmatrix} \tag{40}$$

the effects (electric and magnetic polarization, chirality and reciprocity) are separated in the constitutive relations (1), (2):

$$\begin{pmatrix} \bar{D} \\ \bar{B} \end{pmatrix} = \begin{pmatrix} \epsilon & (\chi - j\kappa)\sqrt{\mu_0\epsilon_0} \\ (\chi + j\kappa)\sqrt{\mu_0\epsilon_0} & \mu \end{pmatrix} \begin{pmatrix} \bar{E} \\ \bar{H} \end{pmatrix} \tag{41}$$

For a lossy material, all these four parameters can be complex for time-harmonic excitation of the fields.

In the other systems of constitutive relations that have been briefly discussed in this communication, the parameters have been expressed as functions of the material parameters of (1), (2). It is worth noting that in all these transation formulas, always the permittivity, permeability, and nonreciprocity are even functions of chirality (given in one of the other systems). This means that these three parameters are equal for a material and its mirror image (which is a medium possessing κ with a change in sign with respect to the original medium). Also κ is an odd function of all chiralities of the other notations, which also agrees with intuition.

Not so intuitive is that also nonreciprocity behaves in a similar way, a fact that is reflected by the similar position of χ compared to κ in the expressions above. In other words, also the permittivity, permeability, and chirality are even functions of nonreciprocity. And again in the translation formulae, nonreciprocity is itself always an odd function of the nonreciprocity parameter of the other notations.

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THEORY OF GUIDED WAVE IN CHIRAL MEDIA : A STEP FOR MEASURING CHIRAL MATERIAL PARAMETERS

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The constitutive relations of isotropic lossy chiral composite consisting of chiral objects for time-harmonic fields exp(-iωt) have the form $\mathbf{D} = \varepsilon_c\mathbf{E} + i\xi_c\mathbf{B}$ and $\mathbf{H} = i\xi_c\mathbf{E} + \mathbf{B}/\mu_c$ where $\varepsilon_c = \varepsilon_0(\varepsilon' + i\varepsilon'')$, $\mu_c = \mu_0(\mu' + i\mu'')$ and $\xi_c = \xi_c' + i\xi_c''$ represent complex permittivity and permeability, and chirality admittance, which is a measure of the handedness of the medium. These materials possess two bulk eigenmodes of propagation, a right circularly polarized (RCP) and a left circularly polarized (LCP) plane wave with two differing complex wavenumbers. Recently, the guided wave propagation in chiral media has been the subject of intense research. Among those studies we can mention the introduction by P. Pelet and N. Engheta [1] of *chirowaveguides* which consist of cylindrical wave-guiding structures filled with isotropic lossless chiral materials.

In this talk, we review all the results of our recent theoretical study on parallel-plate waveguide partially filled with isotropic chiral materials (loss or lossless) [2-4]. First, we present an overview of the theory of chirowaveguides. Secondly, we analyse the study of the canonical problem of reflection and transmission of guided modes at an air-chiral interface and at a lossless chiral slab transversely located in a parallel-plate waveguide. In those two problems, the chiral materials were assumed to be lossless; now we theoretically study the effect of lossy chiral materials on guided modes in such waveguides and furthermore we analyze, first, the reflection and transmission of guided modes at a lossy chiral slab *(case a)* and secondly the reflection of a lossy chiral slab backed by a perfect conductor *(case b)*. The motivation behind this study is the potential application of this problem in the design of novel measurement techniques for determining material complex parameters of lossy chiral composites.

So we have studied the effect of chirality and then the influence of lossy materials in all these problems. The results are the following. For our analysis on reflection and transmission of guided electromagnetic waves at an achiral-chiral (lossless) interface in a parallel-plate waveguide, it is found that in order to satisfy the boundary conditions at the achiral-chiral interface, the reflected and transmitted waves need to be hybrid. For lossy materials, we first present the results of our analysis on the dispersion relations, Brillouin diagrams, cut-off frequencies, propagation and attenuation coefficients of the modes. It must be noted that for lossy chirowaveguide, the dispersion curves approach the line k_+ or k_- according to the chiral parameters ε_c and μ_c. Secondly, we have obtained the reflected and transmitted modes (only *case b*) and have evaluated the effect of chirality and the loss of the medium on power of reflected and transmitted waves. Finally we address rules to solve the inverse problem for the potential applications in material characterizations. Electromagnetic shielding and design of devices and components could be also potential applications of these studies.

In this talk, we present also briefly other Microwave Chirality Research at CEA-CESTA : First, modelling of heterogeneous chiral materials by the calculation of electromagnetic scattering of a chiral element (thin helix) and secondly the design of chiral shields (preparation of chiral samples and free space measurements) [5-6].

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Measuring electrical, magnetic, and chiral material parameters

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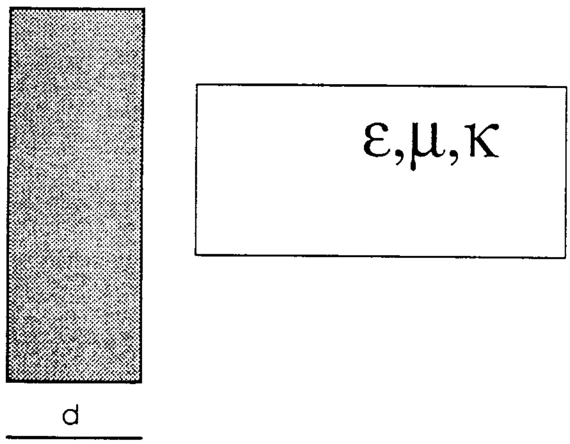
Problem

- unknown plane slab

- what are the electrical material parameters



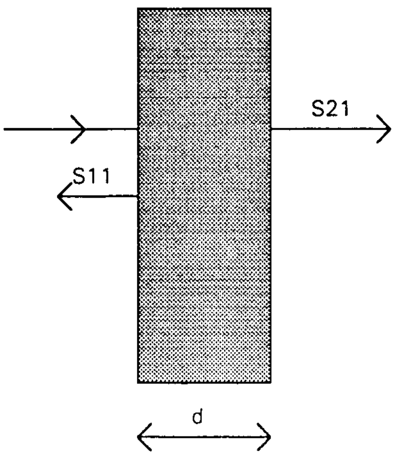
Measuring traditional, nonchiral, material

- two complex parameters ε,μ

- measuring methods:
    - resonator methods
        - ε or μ
        - low lossy materials

    - reflection / transmission methods
        - transmission line methods
        - free space methods

Reflection/Transmission methods

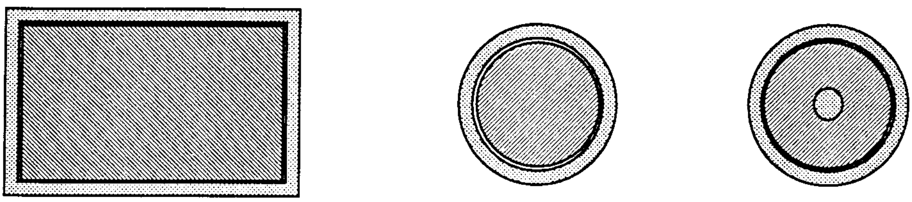


- measurement of the reflected and transmitted field

- measurements are done with well calibrated network analyzer

Transmission line method

- sample inside the waveguide
- preparation of the sample is difficult
- gaps between sample and waveguide make results inaccurate

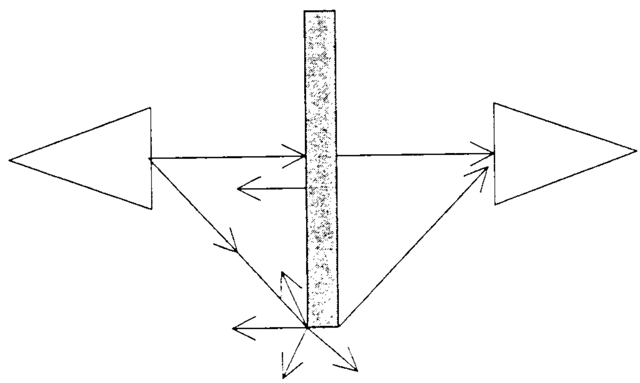


<u>Free space methods</u>

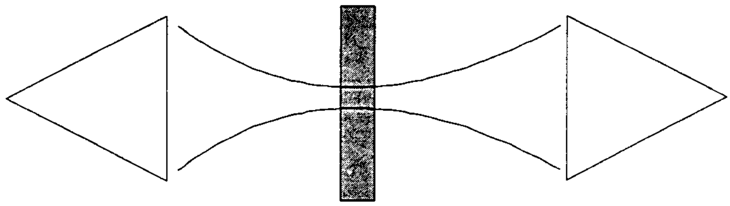
- purpose to measure reflected and transmitted signals in free space

- no preparation of small samples

- problem: diffraction from the edges of the slab



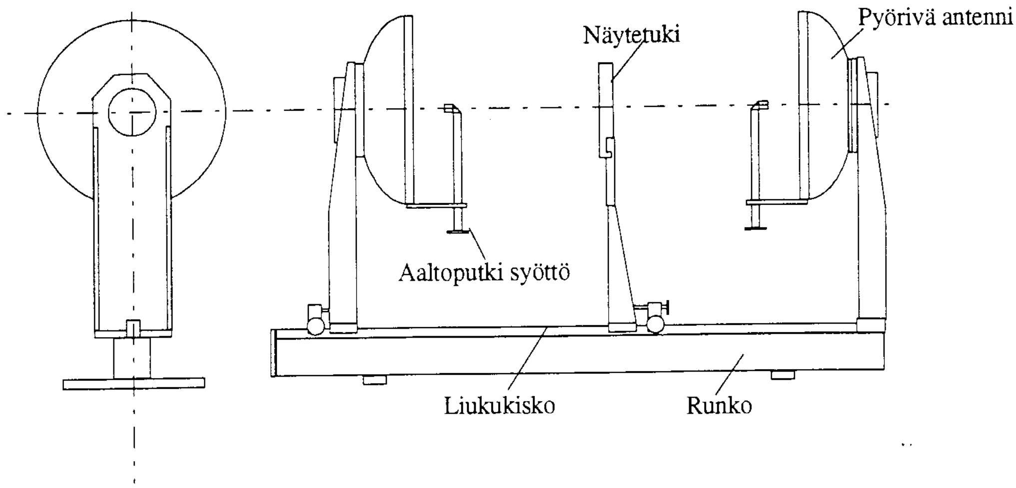
- focusing antennas

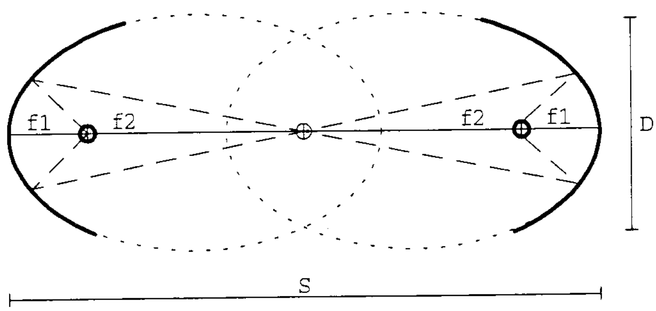


- no edgediffractions

-calibration with TRL-method

- time domain gating to avoid multible reflections


Näytetuki
Pyörivä antenni
Aaltoputki syöttö
Liukukisko
Runko


f1
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S

Calculating material parameters from measured S-parameters

- measured S11- and S21-parameters

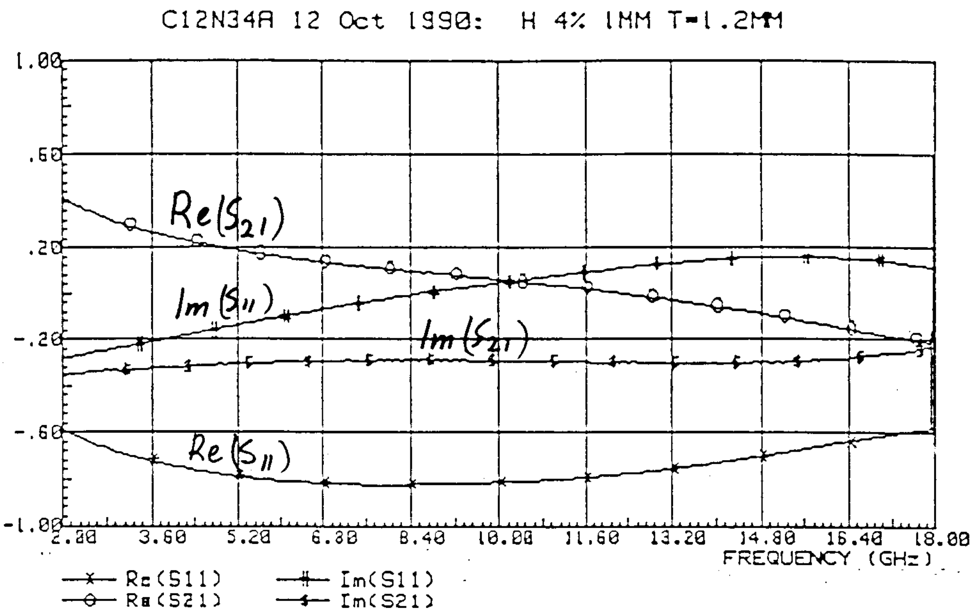
$$K = \frac{S_{11}{}^2 - S_{21}{}^2 + 1}{2S_{11}}$$

$$\Gamma = K \pm \sqrt{K^2 - 1}$$

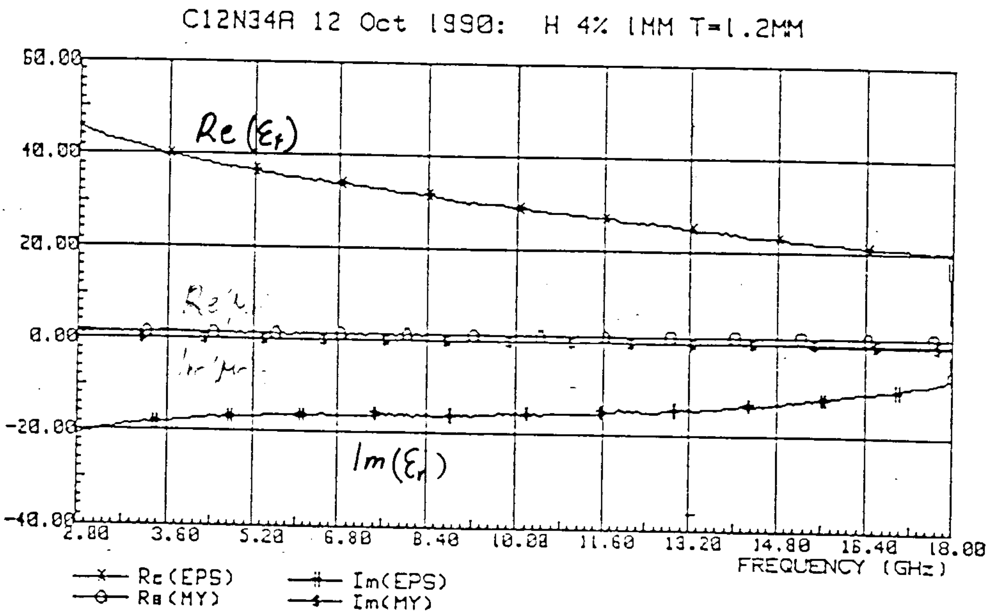
$$T = \frac{S_{11} + S_{21} - \Gamma}{1 - (S_{11} + S_{21})\Gamma}$$

$$k = \frac{j}{d}(\ln(T) + n2\pi) \quad \text{n} = 0, \pm 1, \pm 2, \ldots$$

$$\boxed{\begin{aligned} \varepsilon_r &= \frac{1-\Gamma}{1+\Gamma}\frac{k}{k_0} \\ \mu_r &= \frac{1+\Gamma}{1-\Gamma}\frac{k}{k_0} \end{aligned}}$$



Kuva 2.1 Esimerkki perusaineelle mitatuista S-parametreistä (hiilen osuus 4%, näytteen paksuus 1.2mm).



Kuva 2.2 Mitatuista S-parametreista (kuva 2.1) EPSMY-ohjelmalla lasketut suhteelliset permittiivisyys- (e_r) ja permiabiliteettikäyrät (μ_r).

Measuring the chiral material

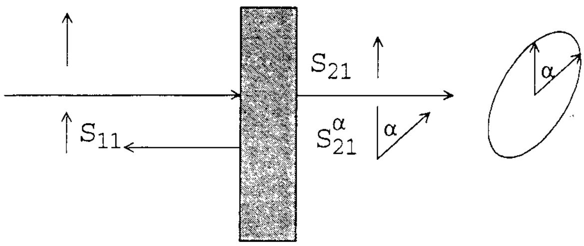
- three complex parameters ε,μ and κ $\Rightarrow$ more measurements are needed

- polarization of the wave is changed when the wave goes throught the chiral slab

- electrical field behind the slab

$$\vec{E}_t = E_t\left(\vec{u}_x + \tan(-k_0 d\kappa)\vec{u}_y\right)$$

- lets measure the polarization properties of the transmitted field



- transmitter and receiver antennas linearly polarized

$$G = \frac{S_{21}^{\alpha} - S_{21}\cos(\alpha)}{S_{21}\sin(\alpha)} \qquad G = \frac{AR\sin(\tau) - j\cos(\tau)}{AR\cos(\tau) + j\sin(\tau)}$$

$$K = \frac{S_{11}^{\ 2} - S_{21}^{\ 2}(1+G^2)+1}{2S_{11}}$$

$$\Gamma = K \pm \sqrt{K^2 - 1}$$

$$T = \frac{S_{11} + S_{21}\sqrt{(1+G^2)} - \Gamma}{1 - (S_{11} + S_{21}\sqrt{(1+G^2)})\Gamma}$$

$$k_c = \frac{j}{d}(\ln(T) + n2\pi)$$

n = 0, ±1, ±2,...

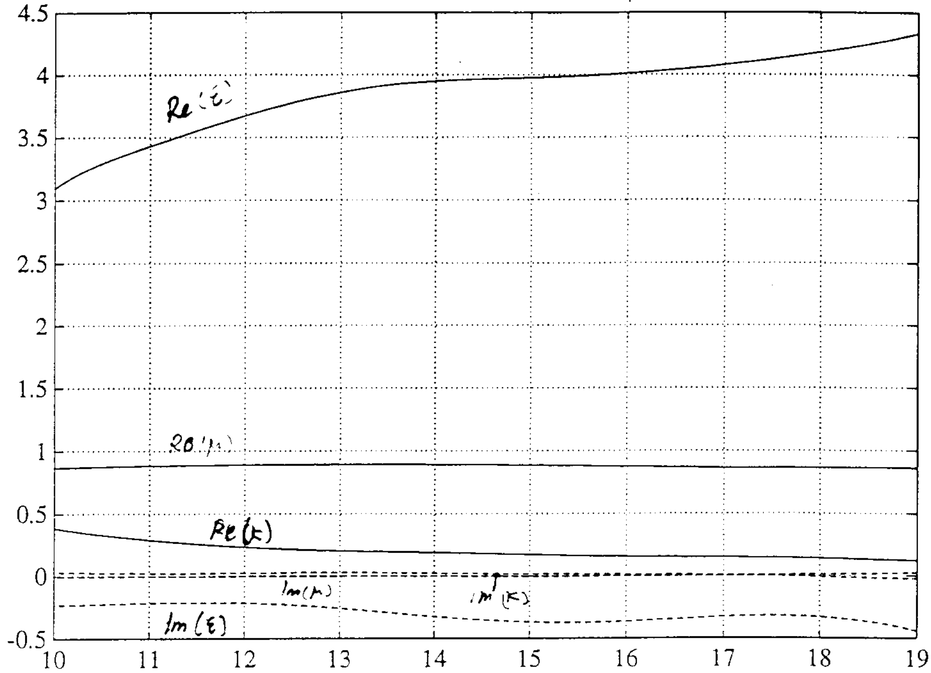
$$\varepsilon_c = \frac{1-\Gamma}{1+\Gamma}\frac{k_c}{k_0}$$

$$\mu_c = \frac{1+\Gamma}{1-\Gamma}\frac{k_c}{k_0}$$

$$\kappa = \frac{-(\arctan(G)+m\pi)}{k_0 d}$$

m = 0, 1, 2,...

Suhteellinen epsilon, myy ja kappa, Levy 4, alfa=-50



Taajuus / GHz

Taajuus	Re(εr)	Im(εr)	Re(μr)	Im(μr)	Re(κ)	Im(κ)
1,00E+10	3,12	-0,23	0,87	0,00	0,38	0,03
1,10E+10	3,43	-0,22	0,88	0,00	0,29	0,02
1,20E+10	3,67	-0,22	0,89	0,00	0,23	0,03
1,30E+10	3,85	-0,26	0,89	0,00	0,20	0,03
1,40E+10	3,95	-0,32	0,89	0,00	0,19	0,02
1,50E+10	3,98	-0,37	0,88	0,00	0,17	0,02
1,60E+10	4,01	-0,37	0,87	0,00	0,16	0,02
1,70E+10	4,08	-0,33	0,86	0,01	0,15	0,01
1,80E+10	4,18	-0,33	0,86	0,01	0,14	-0,01
1,90E+10	4,27	-0,39	0,86	0,02	0,13	-0,02

emk440

Electromagnetic Properties of Lossless n-Turn Helices in the Quasi-Stationary Approximation

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1 Introduction

The study of electromagnetic (EM) properties of bi-isotropic (BI) media at UMIST is comparatively new. It is the subject of the first author's PhD Project in Applied Electronics. The project aims to be a '3M' approach:-

- *modelling* of chiral and BI media: this includes the fundamental study of the physical properties of general BI media, as well as the properties of specific chiral structures that may be implemented at microwave frequencies;
- *manufacturing* of chiral and BI media: advances in the tailoring and practical implementation of suspended chiral microstructures are sought;
- *measurement* of chiral and BI media at microwave frequencies: because of the complexity of the EM effects, new measurement techniques are considered.

Advances in all three areas have been made in the recent past. In this presentation, we will discuss one aspect, *ie* the modelling of practical chiral structures. The EM properties of an n-turn helix will be derived in the quasi-stationary approximation, its performance in terms of EM activity will be predicted and compared with published data.

2 Field Induced by an n-Turn Helix

Chiral properties of ideal, short-wire helices, shown in Fig 1, have been derived by Jaggard *et al.* The helical structure under consideration here is sketched in Fig 2. The spring has n turns, free length $2l$, external diameter $2a$, gauge diameter $2b$ and pitch $p = \frac{2l}{n}$. It is made of perfectly conducting wire and embedded in a lossless medium. It is assumed that $2l \gg 2a$ so that fringing of the magnetic field may be neglected.

An external plane wave, characterized by $\{\underline{E}_e, \underline{H}_e\}$, is incident onto the helix. After some calculations, it is found that the induced electric field $\underline{E}_i$ is related to the induced current I_i by:-

$$E_i \propto \frac{n}{4l}\, r \left(-\frac{\partial I_i}{\partial t}\right), \qquad r \leq a \tag{1}$$

$$\propto \frac{n}{4l}\,\frac{a^2}{r} \left(-\frac{\partial I_i}{\partial t}\right), \qquad r \geq a \tag{2}$$

and is directed tangential to concentric circles around the helical axis. Thus, the induced electric field has a constant amplitude on concentric circles around the helical axis, it increases linearly with distance inside the helix and decreases as $\frac{a^2}{r}$ outside the helix.
This induced field affects the field scattered by the helix. In a composite medium of v helix vol pct springs embedded in a dielectric host, this causes the interaction between springs to be much larger than in a composite medium of the same vol pct v of nonchiral conducting particles. This explains why the EM properties of dielectrics with embedded microsprings change drastically at a threshold $v = v_o$ which is much lower than the classical 10% limit used in multiple scattering theory [2].

3 Chiral Admittance and EM Rotation for an n-Turn Helix

EM activity caused by chiral objects is basically an interaction between $\underline{E}$ and $\underline{H}$, *ie* a coupling between the capacitance C_n and inductance L_n of a collection of chiral objects. In the quasi-static approximation, the helices are very small as seen by the incident wave, hence the wave impedance of the chiral medium may then be calculated from the lumped impedances of the helices.
Compared to the idealized short helix model, inclination of the windings as well as interaction between them needs to be accounted for in the model. This results in an expression for the chiral admittance of a single round-wire helix:-

$$\xi_c = \sqrt{\frac{\epsilon}{\mu}}\frac{2l\,(\pi a)^2}{(n-1)\,\log\left(\frac{2l}{nb\,\sqrt{1+\left(\frac{l}{\pi n a}\right)^2}} - 2\right)} \tag{3}$$

in which ϵ, μ denote the permittivity and permeability of the host medium, respectively. With appropriate averaging over the (random) orientation of the helices and their concentration N, this allows the resonance frequency f_r as well as the chiral admittance ξ_c at resonance to be calculated [1]. The results are listed in Table 1, in which the computed EM rotation is compared with previously measured data. The subscripts 1 and N refer to a single helix and N helices/m^3, respectively.

Quantity	Unit	Ref 2	Ref 3
ϵ'_{host}	–	2.5 (at 10 GHz)	2.95 (at 7 GHz)
Helix Concentration N	$10^6/\mathrm{m}^3$	139.8	36.7
	vol %	3.2	0.8
L_n	nH/m	5.32	6.06
C_n	fF/m	93.7	89.4
$f_r = \frac{1}{2\pi\sqrt{L_n C_n}}$	GHz	7.13	6.84
$(\xi_c)_1$	pS m^3	9.13	9.43
$(\phi_t(f = f_r))_1$ (computed)	μdeg	.074	.100
$(\xi_c)_N$	μS m^3	425.4	115.3
$(\phi_t(f = f_r))_N$ (computed)	deg	42.27	12.88
$(\phi_t(f = f_r))_N$ (measured) (approx)	deg	NA	5 - 15

Table 1: Comparison between Predicted and Measured Data

Given the fact that the information on the host material and helices is insufficient for accurate calculations at f_r (and, therefore, ϕ_t), and taking into account that the computed values for ϕ_t serve as a lower limit due to multiple scattering, it may be concluded that the model is sufficiently accurate to provide a good estimate of the EM rotation by the sample.

Finally, Fig 3 shows the dependence of the EM activity of a single helix on its dimensions, which have been normalized with respect to the helix length.

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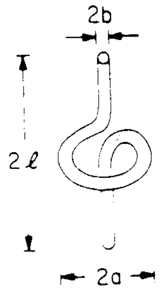

Figure 1

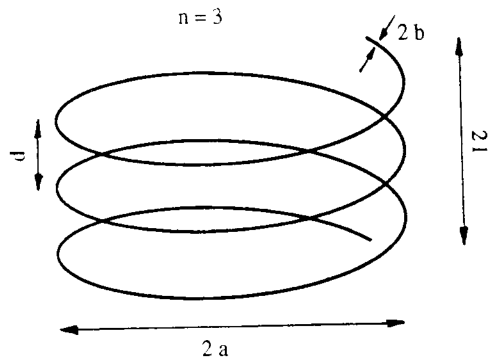

Figure 2

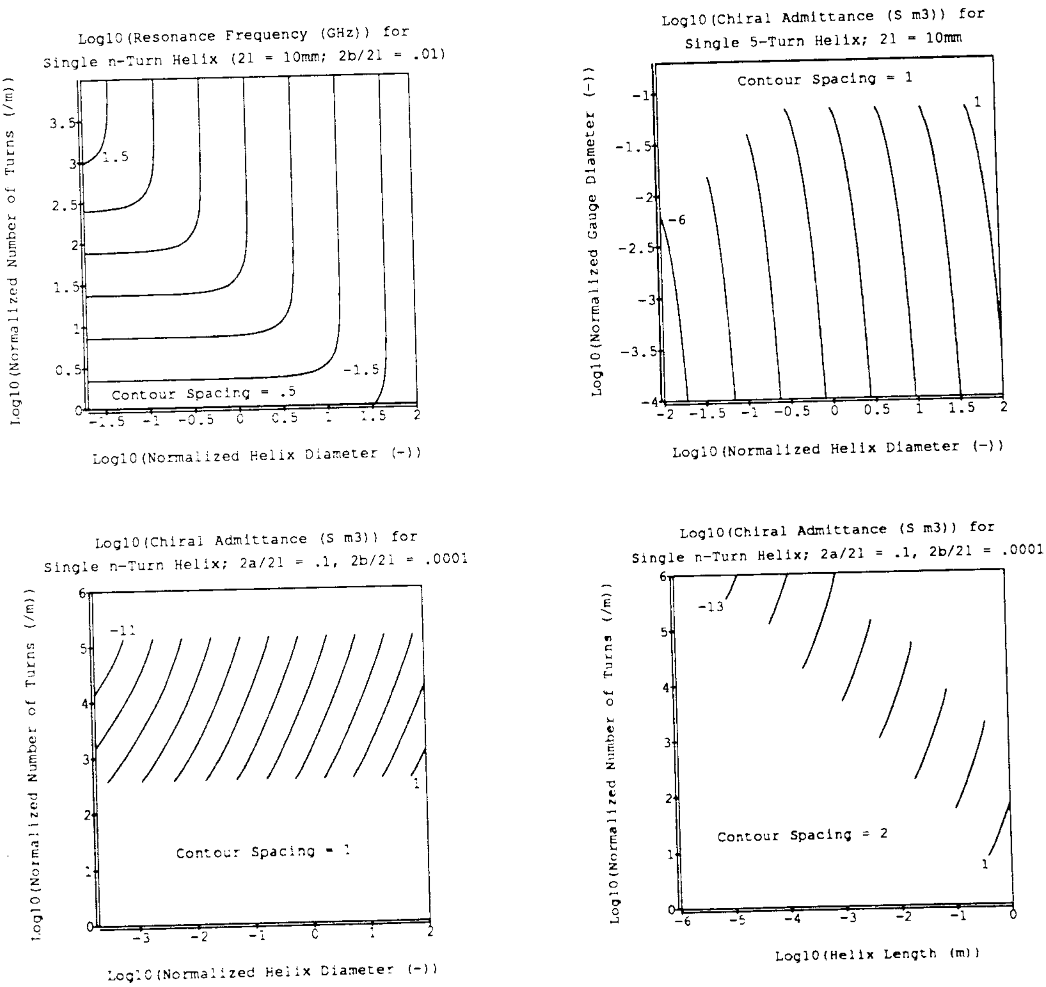

Figure 3

Electromagnetics Laboratory Report 128, Helsinki University of Technology
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Plane wave propagation in a uniaxial bianisotropic medium

Ismo V. Lindell and Ari J. Viitanen

Report 128
November 1992

Abstract
Uniaxial bianisotropic medium is a generalization of the bi-isotropic and chiral media which recently have been subject to intensive research. Such a medium results, for example, when microscopic helices with parallel axes are positioned in a host dielectric in random locations. Plane wave propagation in such a medium is studied and a simple solution for the dispersion equation is found. Numerical examples for the wave number surfaces of the medium are displayed.

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Electromagnetics Laboratory Report 132, Helsinki University of Technology
- available on request from the authors

Plane wave propagation in a uniaxial bianisotropic medium with an application to a polarization transformer

Ari J. Viitanen and Ismo V. Lindell

Report 132
January 1993

Abstract
Uniaxial bianisotropic medium is a generalization of the well-studied bi-isotropic and chiral media. It is obtained, for example, when microscopic helices with parallel axes are positioned in a host dielectric in random locations. Plane wave propagation in such a medium is studied and a simple solution for the dispersion equation and for the eigenwaves are found. As a numerical example, polarization properties of a transverse wave propagating in a uniaxial bianisotropic medium is considered. The results give a simple possibility to construct a polarization transformer with a transversely uniaxial chiral medium for changing the polarization of a propagating plane wave.

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Electromagnetics Laboratory Report 133, Helsinki University of Technology
- available on request from the authors

Plane-wave propagation in a transversely bianisotropic uniaxial medium

Ismo V. Lindell, Ari J. Viitanen, Päivi K. Koivisto

Report 133
January 1993

Abstract

Transversely bianisotropic uniaxial medium considered in the present paper can be obtained, e.g., by mixing metal helices with an isotropic base medium in such a way that the axes of the helices are randomly oriented but perpendicular to a fixed direction in space. The medium is a generalization of the well-studied chiral medium and somewhat similar to the recently studied axially bianisotropic uniaxial medium, which has interesting polarization properties. Plane wave propagation in the medium is studied and the solution for the dispersion equation is given. Numerical examples for the wave numbers corresponding to the two eigenwaves of the medium are displayed. It is seen that, unlike in the axially bianisotropic uniaxial medium, there are no optical axes in the present medium, in general.

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Novel uniaxial bianisotropic materials

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Recently, a novel concept of artificial composite materials with Ω-shaped metal elements was introduced in [1]. The idea originated from wide and intensive studies of isotropic chiral media and their electromagnetic properties in microwave regime. A typical chiral inclusion — a small wire helix — can be considered as an electric dipole connected with a magnetic dipole in such a fashion that high-frequency electric field induces a magnetic field parallel to the original electric field component, and *vice versa*. This causes optical or microwave activity - rotation of the polarization plane of a linearly polarized propagating wave. Such chiral structures most effectively interact with circularly polarized waves, since the right and left circular polarizations are eigenpolarizations of waves in unbounded biisotropic media. However, one may suggest other geometrical configurations which can provide stronger wave-material interaction in other circumstances we deal with in microwave engineering. One of such modifications was introduced in [1], where it was suggested to use particles shaped like the capital Greek letter Ω instead of chiral helical particles to ensure first-order effect on the propagation factor in a partially filled rectangular waveguide. The material can have other interesting applications [2]. In a regular microstructure with Ω-shaped conductive inclusions, there exists additional interaction between electric and magnetic fields which lay in orthogonal planes, and the material can be modelled by bianisotropic constitutive equations.

Here we advance an idea of another modification of such microstructure configurations which can be better suited for use in plane non-reflecting coverings and antenna radomes.

We focus the analysis on plane screens which are designed to interact with linearly polarized electromagnetic waves and suggest a modification which can provide uniform operation for linearly polarized waves with any electric field direction (or for unpolarized plane waves). This can be achieved by introducing a second ensemble of Ω- particles inside the matrix. As a result, the structure will interact more effectively with linearly polarized waves of any polarization direction or with unpolarized plane waves. Such a medium can be named as uniaxial omega-medium, because there exists only one particular direction - that one normal to the interfaces of a layer. The size of Ω-shaped elements is assumed to be smaller than the wavelength, hence the medium can be described by effective averaged material parameters and the material is modelled by uniaxial bianisotropic constitutive relations which couple electric and magnetic fields.

In the report we develop general theory of plane wave propagation in novel media and study reflection and transmission in plane uniaxial bianisotropic layered structures. As an example interesting for applications, we consider in detail reflection from a plane metal surface covered with a lossy layer. The example demonstrates that with the additional Ω-shaped wire elements absorption can be enhanced in a wide frequency range. The additional material parameter can help to manage properties of anti-reflection coatings, in a way similar to the effect provided by the chirality parameter of biisotropic materials. Another example is the reflection and transmission through a plane slab. Here it is seen that the material is perspective for potential use in antenna radomes since a nearly transparent and non-reflecting covering can be designed. Also, lossy slabs can be designed to serve as non-reflecting absorbers.

It appears that the input impedance of a lossy layer on an ideally conducting surface or in free space can be matched with the free-space wave impedance by tuning the additional coupling parameter. The impedance matching condition is frequency-independent (provided the material parameter values can be assumed to be constants) and that suggests superwide frequency band for prospective anti-reflection coverings and antenna radomes.

Designing the material, one can compromise between the coupling parameter value, the losses and the slab thickness. If the impedances match, the thickness is essential for transmission properties, but not for the reflection. Comparing with the requirements known for chiral low- reflection screens, it seems easier to achieve desired effects with the Ω-composites, because in the chiral screens rather high degree of chirality is required.

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Macroscopic predictions of material parameters for heterogeneous bi media

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In electromagnetics, we often treat materials as homogeneous, *i.e.* their material parameter functions are constants with respect to space. These material parameters are macroscopic descriptions that cover all polarization and loss phenomena that take place on the microscopic physical level, and with certain restrictions, these quantities are sufficient in solving electromagnetic wave problems involving matter.

However, strictly taken, no media are homogeneous, at least as one looks at them in sufficiently small scale. There are material boundaries, grain walls, etc. Media are random. Especially, as one tries to "see" (and seeing is measuring the reflection or emission of electromagnetic waves) how the medium looks like, using a wavelength equal to the correlation length of the spatial permittivity function of the medium, the scenery is kaleidoscopic: the wave behaves totally differently than in a homogeneous medium; it does not propagate along straight lines as the optical wave in air but it scatters strongly, it is reflected and refracted.

The macroscopic properties of dielectrically heterogeneous media have been a research topic for physicists for over a century, and studies have resulted in a wealth of mixing theories. But how about the special guests of this Workshop: "exotic" materials, chiral and bi-isotropic media?

Compared to classical dielectric or magnetic materials, it is intuitively clearer that chiral media are heterogeneous. The concept of chirality is so much connected with geometry, the handedness that is present at some scale within the microstructure of the material, that the first thought as one sees a sample of chiral material is to try to find the helical elements responsible for the chirality. An inevitable consequence of the geometric structure is dielectric inhomogeneity in the medium.

Much effort in the chiroelectromagnetic research of these years is being expended on the electromagnetic modelling of a canonical chiral structure, the metal helix. There are studies of what kind of current distribution will be induced on the helix as a plane wave is incident upon it. This is not an easy problem.

In the present talk, however, I will not consider the emergence of chirality through geometry. Rather, in a way of "analytically continuing" the classical dielectric mixture rules and equations, I shall discuss how, given the chirality parameters[1] of the constituents of the mixture, to predict the macroscopic chirality (and also permittivity and permeability) of the whole. In other words, in the mixture under study there are inclusions that are assumed to be of homogeneous bi-isotropic material, and these determine the larger-scale parameters.

In this analysis, the starting point is to find the polarizabilities of simple-shaped bi-isotropic particles. Due to the magnetoelectric coupling in the complex medium, a four-element polarizability matrix, including cross-polarizability terms, is needed. After knowing this, the classical Maxwell-Garnett and other mixing rules can be generalized to cover bi-isotropic media. In appearance, these expand and inflate in this process, and the formulas become coupled: for example, the permittivity of one inclusion affects the nonreciprocity, chirality, and permeability of the mixture.

The results shown in the presentation are collected from my studies on bi-isotropic mixtures, and many illustrations have been published before. Most results can be found in the references [1, 2, 3, 4, 5].

Some general conclusions– The salient features of the mixing process can be seen already from the simplest two-component mixture: bi-isotropic spheres embedded in isotropic background medium. The coupling of all parameters was already mentioned: one macroscopic parameter is a function of all six material parameter quantities of the problem. Secondly, there exist wonderful dualities in the macroscopic material formula expressions. For example, the way the <u>permeability</u> of the inclusion affects the effective permeability, chirality, nonreciprocity, and permittivity, is the same as the way the <u>permittivity</u> of the inclusion affects the effective permittivity, chirality, nonreciprocity, and permeability. Also, the functional dependence of the macroscopic permittivity[2] on the inclusion chirality is exactly the same as on the inclusion nonreciprocity. Put into a more nonredundant form: $\epsilon_m(\epsilon_i, \mu_i, \kappa_i, \chi_i; \epsilon_h, \mu_h) = \epsilon_m(\epsilon_i, \mu_i, \chi_i, \kappa_i; \epsilon_h, \mu_h)$. Further; the macroscopic chirality depends on inclusion nonreciprocity identically with macroscopic nonreciprocity de-

[1]and also nonreciprocity (fully bi-isotropic) parameters

[2]The same applies for the macroscopic permeability.

pendence on inclusion chirality.

An essential observation is the fact that the effective (=macroscopic) permittivity and permeability of a mixture are even functions of both the chirality and nonreciprocity of the component material. Hence, firstly, the sign of handedness, *i.e.* whether left or right handed, should not have effect on these parameters, and they are true scalars, invariant of spatial inversion. This is obvious: samples of media that are mirror images of one another should have the same permittivity and the same permeability. But such is also the dependence on the sign of the inclusion noreciprocity parameter, although the intuitive support for this is not as evident. Also the effective chirality is an odd function of the chirality of the inclusion material (and a pseudoscalar): changing the handedness of the component changes the handedness of the mixture. (And again, a similar conclusion holds for the effective nonreciprocity.)

Furthermore, due to the fact that these functional dependences are even, perturbation expansions start with the second-power term, and therefore the effect for small chiralities (and nonreciprocities) is small. In other words, the chirality of the inclusion phase has little effect on the macroscopic permittivity and permeability, at least for high permittivity and permeability contrasts between the inclusion and background phases. On the other hand, naturally the chirality of the inclusion is the dominant parameter defining the effective chirality of the mixture. Finally, the effective chirality of a mixture is decreased by high permittivity and/or permeability of the inclusion phase.

How, then, about inclusions of other shapes? The only other forms of inclusions whose polarizabilities can be solved in closed form are ellipsoids. By using chiral inclusions of ellipsoidal shapes, a further range of mixture parameters can be tailored. The effects vary: by using needle-shaped or disk-shaped inclusions, larger effective parameters can be achieved, although the shape-effect depends on the dielectric and magnetic contrast between the inclusion and host phases. One observation is, however, always valid: spherical inclusions produce minimum effects in the macroscopic properties, and each deviation from this extremum shape always increases the value achieved by the spherical geometry.

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Covariant methods in the theory of electromagnetic waves

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This presentation focused on the history and present state of crystal optics, starting from the basic works of Pockels (1906), Drude (1912), and Born (1932). The different crystal types were classified according to the symmetry and other properties of the corresponding polarizability matrices. Optical activity and gyrotropy in crystals has received considerable attention starting from late 1950's in Belorussia and the former Soviet Union in general. The talk discussed in detail the historical development of the constitutive relations of optically active media, as the early suggestions by Drude and Born were shown to be inconsistent with the energy conservation principle. The material equations in the final form are nowadays labeled often *Drude–Born–Fedorov*-relations in the Western literature. Special emphasis in the talk was given to the coordinate-free covariant representation of quantities that appear in the electromagnetic analysis of anisotropic and bianisotropic media.

(from the notebook of the Workshop organizer)

Fundamental principles resticting the macroscopic phenomenological description of matter

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Time-Reversal Symmetry
(Onsager-Casimir Principle)

F_a, X_a $\qquad X_a = \sum_b K_{ab} F_b \qquad \dot{W} = \sum_a F_a \dot{X}_a$

$$\boxed{K_{ab} = \eta_a \eta_b K_{ba}}$$

where $\eta_a = +1$ or $\eta_a = -1$ in connection with time-revers operation $T(t \to -t)$: $\; T F_a = \eta_a F_a$

Macroscopical Linar Electrodynamics

$$(F_a) = \begin{pmatrix} \underline{E} \\ \underline{H} \end{pmatrix}, \quad (X_a) = \begin{pmatrix} \underline{D} \\ \underline{B} \end{pmatrix} \qquad \left\{ \boxed{\begin{array}{l} \underline{D} = \varepsilon \underline{E} + \alpha \underline{H} \\ \underline{B} = \beta \underline{E} + \mu \underline{H} \end{array}} \right. \qquad \dot{W} = \underline{E}\dot{\underline{D}} + \underline{H}\dot{\underline{B}}$$

$$(K_{ab}) = \left(\begin{array}{c:c} \varepsilon & \alpha \\ \hdashline \beta & \mu \end{array}\right) \quad a, b = 1, 2, \dots, 6 \qquad T\underline{E} = \underline{E}, \; T\underline{H} = -\underline{H},$$

$$\eta_{1,2,3} = +1, \; \eta_{4,5,6} = -1$$

$$\left.\begin{array}{l} K_{12} = K_{21} \\ K_{13} = K_{31} \\ K_{23} = K_{32} \end{array}\right\} \text{or} \quad \left.\begin{array}{l} \varepsilon_{12} = \varepsilon_{21} \\ \varepsilon_{13} = \varepsilon_{31} \\ \varepsilon_{23} = \varepsilon_{32} \end{array}\right\} \qquad \left.\begin{array}{l} K_{45} = K_{54} \\ K_{46} = K_{64} \\ K_{56} = K_{65} \end{array}\right\} \text{or} \quad \left.\begin{array}{l} \mu_{12} = \mu_{21} \\ \mu_{13} = \mu_{31} \\ \mu_{23} = \mu_{32} \end{array}\right\}$$

$$\left.\begin{array}{l} K_{14} = -K_{41} \\ K_{15} = -K_{51} \\ K_{16} = -K_{61} \\ K_{24} = -K_{42} \\ \dots \end{array}\right\} \text{or} \quad \left.\begin{array}{l} \alpha_{11} = -\beta_{11} \\ \alpha_{12} = -\beta_{21} \\ \alpha_{13} = -\beta_{31} \\ \alpha_{21} = -\beta_{12} \\ \dots \end{array}\right\}$$

$$\boxed{\varepsilon_{ij} = \varepsilon_{ji}} \quad \boxed{\mu_{ij} = \mu_{ji}} \quad \boxed{\alpha_{ij} = -\beta_{ji}} \qquad \boxed{\begin{array}{l} \varepsilon = \tilde{\varepsilon} \\ \mu = \tilde{\mu} \\ \alpha = -\tilde{\beta} \end{array}}$$

If $K_{ab} = K_{ab}(\ell_N)$ where $\ell_1, \ell_2, \dots$ are ext. or int. parameters, we have

$$K_{ab}(\ell_N) = \eta_a \eta_b K_{ba}(T\ell_N) \qquad \left| \begin{array}{l} \{\ell_N\} = \{\underline{e}, \underline{h}\} \\ T\underline{e} = \underline{e}, \; T\underline{h} = -\underline{h} \end{array}\right. \qquad \boxed{\begin{array}{l} \varepsilon(\underline{e}, \underline{h}) = \tilde{\varepsilon}(\underline{e}, -\underline{h}) \\ \mu(\underline{e}, \underline{h}) = \tilde{\mu}(\underline{e}, -\underline{h}) \\ \alpha(\underline{e}, \underline{h}) = -\tilde{\beta}(\underline{e}, -\underline{h}) \end{array}}$$

Space Symmetry of Media
(Cryst. Symmetry)

$$\mathrm{rot}\,\underline{E} = -\frac{1}{c}\frac{\partial \underline{B}}{\partial t} \qquad \underline{D} = \varepsilon \underline{E} + \alpha \underline{H}$$

$$\mathrm{rot}\,\underline{H} = \frac{1}{c}\frac{\partial \underline{D}}{\partial t} + \frac{4\pi}{c}\underline{j} \qquad \underline{B} = \beta \underline{E} + \mu \underline{H}$$

$$\underline{r}' = U\underline{r} \rightarrow \begin{cases} \underline{D}' = U\underline{D}, & \underline{B}' = |U|U\underline{B} \\ \underline{E}' = U\underline{E}, & \underline{H}' = |U|U\underline{H} \end{cases} \qquad \boxed{\begin{matrix} \varepsilon' = U\varepsilon\tilde{U} & \alpha' = |U|U\alpha\tilde{U} \\ \mu' = U\mu\tilde{U} & \beta' = |U|U\beta\tilde{U} \end{matrix}}$$

If $U = S$ is a matrix of symmetry transformation of crystal:

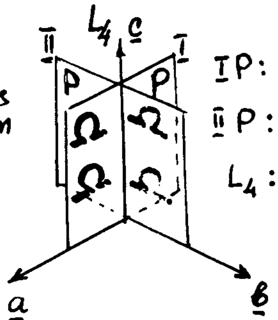
$$\boxed{\begin{matrix} \varepsilon = S\varepsilon\tilde{S} \\ \mu = S\mu\tilde{S} \end{matrix}} \qquad \boxed{\begin{matrix} \alpha = |S|S\alpha\tilde{S} \\ \beta = |S|S\beta\tilde{S} \end{matrix}}$$

18 from **32** classes of crystals have $\alpha \neq 0$ $(\beta \neq 0)$

F. Fyodorov (1958): **3** of them (planal cl.) have antisym. α

$$\tilde{\alpha} = -\alpha \qquad (\alpha_{ij} = -\alpha_{ji})$$

Class 4mm



I P: $\underline{a} \to \underline{a}$, $\underline{b} \to -\underline{b}$, $\underline{c} \to \underline{c}$, $|S| = -1$

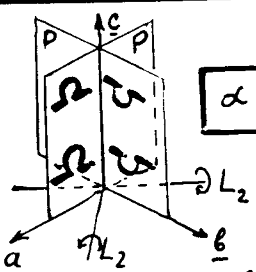
II P: $\underline{a} \to -\underline{a}$, $\underline{b} \to \underline{b}$, $\underline{c} \to \underline{c}$, $|S| = -1$

L_4: $\underline{a} \to \underline{b}$, $\underline{b} \to -\underline{a}$, $\underline{c} \to \underline{c}$, $|S| = +1$

$$\boxed{\varepsilon = \varepsilon_\perp + (\varepsilon_\parallel - \varepsilon_\perp)\underline{c}\cdot\underline{c}}$$

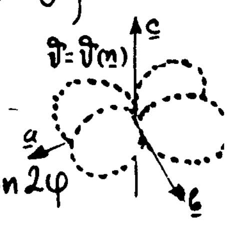
$$\boxed{\alpha = \alpha_{12}(\underline{a}\cdot\underline{b} - \underline{b}\cdot\underline{a}) = -\alpha_{12}\underline{c}^\times} \qquad \alpha = \begin{pmatrix} 0 & \alpha_{12} & 0 \\ -\alpha_{12} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Class $\bar{4}2m$



$$\boxed{\alpha = \alpha_{12}(\underline{a}\cdot\underline{b} + \underline{b}\cdot\underline{a})} \qquad \alpha = \begin{pmatrix} 0 & \alpha_{12} & 0 \\ \alpha_{12} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Rotation of the plane of polarisation

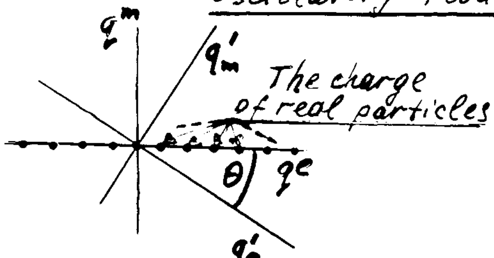


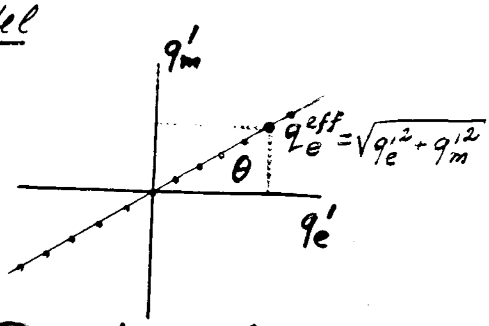
4mm: $\vartheta = k\,\underline{n}\alpha\underline{n} = 0$

$\bar{4}2m$: $\vartheta = k\,\underline{n}\alpha\underline{n} = 2k\alpha_{12}\,\underline{n}\underline{a}\cdot\underline{n}\underline{b} = k\alpha_{12}\sin\theta\cdot\sin 2\varphi$

Duality Transformation and Cinetical Symmetry

Oscillatory Model





Ordinary Maxwell Electrodynamics

$$\begin{cases} \operatorname{rot} \underline{E} = -\dfrac{\partial B}{\partial t} \\ \operatorname{rot} \underline{H} = \dfrac{\partial \underline{D}}{\partial t} + \underline{j}_e \end{cases}$$

$$\begin{cases} \operatorname{div} \underline{B} = 0 \\ \operatorname{div} \underline{D} = \rho_e \end{cases}$$

$$\begin{cases} \underline{D} = \varepsilon \underline{E} + \alpha \underline{H} \\ \underline{B} = \beta \underline{E} + \mu \underline{H} \end{cases}$$

$$\underline{F}^{Lor.} = q_e \underline{E} + q_e [\underline{v}\, \underline{B}]$$

Two charge (dual symmetric) Electrodynamics

$$\begin{cases} \operatorname{rot} \underline{E}' = -\dfrac{\partial \underline{B}'}{\partial t} - \underline{j}'_m \\ \operatorname{rot} \underline{H}' = \dfrac{\partial \underline{D}'}{\partial t} + \underline{j}'_e \end{cases}$$

$$\begin{cases} \operatorname{div} \underline{B}' = \rho'_m \\ \operatorname{div} \underline{D}' = \rho'_e \end{cases}$$

$$\begin{cases} \underline{D}' = \varepsilon' \underline{E}' + \alpha' \underline{H}' \\ \underline{B}' = \beta' \underline{E}' + \mu' \underline{H}' \end{cases}$$

$$\underline{F}^{Lor.} = q'_e \underline{E}' + q'_m \underline{H}' + q'_e [\underline{v}\, \underline{B}'] - q'_m [\underline{v}\, \underline{D}'] = \text{dual invariant}$$

$$m\ddot{\underline{r}} = -k\underline{r} - \nu\dot{\underline{r}} + e\underline{E} + g\underline{H}; \qquad \underline{E}, \underline{H} \sim e^{-i\omega t},$$

$$\underline{r} = \frac{e/m}{\Omega^2 - \omega^2 - i\gamma\omega}\underline{E} + \frac{g/m}{\Omega^2 - \omega^2 - i\gamma\omega}\underline{H}, \qquad \Omega = \sqrt{\frac{k}{m}}, \quad \gamma = \frac{\nu}{m}$$

$$\underline{d} = e\underline{r} \qquad \underline{D} = \underline{E} + 4\pi N \underline{d} \qquad \underline{D} = \varepsilon \underline{E} + \alpha \underline{H}$$

$$\underline{m} = g\underline{r} \qquad \underline{B} = \underline{H} + 4\pi N \underline{m} \qquad \underline{B} = \beta \underline{E} + \mu \underline{H}$$

$$\| \varepsilon = 1 + \frac{4\pi N e^2/m}{\Omega^2 - \omega^2 - i\gamma\omega}, \qquad \| \mu = 1 + \frac{4\pi N g^2/m}{\Omega^2 - \omega^2 - i\gamma\omega},$$

$$\| \alpha = \beta = \frac{4\pi N eg/m}{\Omega^2 - \omega^2 - i\gamma\omega}$$

$$T: \underline{e \to e, \quad g \to -g}$$
$$\underline{E} \to \underline{E}, \quad \underline{H} \to -\underline{H}$$
$$\underline{D} \to \underline{D}, \quad \underline{B} \to -\underline{B}$$

$$\boxed{\begin{aligned} \varepsilon(e,g) &= \tilde{\varepsilon}(e,-g) \\ \mu(e,g) &= \tilde{\mu}(e,-g) \\ \alpha(e,g) &= -\tilde{\alpha}(e,-g) \end{aligned}}$$

The Linear Effect of Magnetic Field on the Natural Optical Activity

A. Serdyukov, V. Belyi (1975)

$$\left.\begin{aligned} \underline{D} &= \varepsilon \underline{E} + i\alpha \underline{H} \\ \underline{B} &= \mu \underline{H} - i\alpha \underline{E} \end{aligned}\right\} \longrightarrow \left.\begin{aligned} \underline{D} &= \varepsilon(\underline{h})\underline{E} + i\alpha(\underline{h})\underline{H} \\ \underline{B} &= \mu(\underline{h})\underline{H} - i\tilde{\alpha}(-\underline{h})\underline{E} \end{aligned}\right\}$$

$$\varepsilon(\underline{h}) = \varepsilon + i\gamma \underline{h}^{\times} \qquad \mu = 1$$

$$\alpha(\underline{h}) = \alpha + i\xi \underline{h}^{\times}$$

$$\underline{E}, \underline{H}, \underline{D}, \underline{B} \sim \exp[i(\underline{k}\underline{r} - \omega t)] \qquad \begin{cases} \underline{m}^{\times}\underline{E} = \underline{B} \\ \underline{m}^{\times}\underline{H} = -\underline{D} \end{cases}$$

$$\underline{k} = \underline{m}\frac{\omega}{c}$$

$$\begin{cases} (\underline{m} - \xi\underline{h})^{\times}\underline{E} + i\alpha\underline{E} = \underline{H} \\ (\underline{m} - \xi\underline{h})^{\times}\underline{H} + i\alpha\underline{H} = -\varepsilon\underline{E} - i\gamma\underline{h}^{\times}\underline{E} \end{cases}$$

$$\underline{m} = n\underline{n} \qquad \underline{n_{\pm} = \sqrt{\varepsilon} \pm \left(\alpha + \frac{\gamma}{2\sqrt{\varepsilon}}\underline{n}\underline{h}\right) + \xi\underline{n}\underline{h}}$$

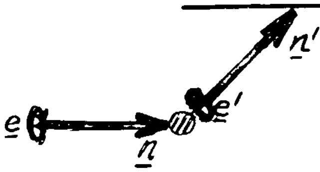
The Moving Medium and the Cinetical Symmetry

$$\begin{cases} \underline{D} = \varepsilon\underline{E} \\ \underline{B} = \mu\underline{H} \end{cases} \longrightarrow \begin{cases} \underline{D} = \varepsilon\underline{E} + \frac{\varepsilon-\mu}{c}\underline{v}^{\times}\underline{H} \\ \underline{B} = \mu\underline{H} - \frac{\varepsilon-\mu}{c}\underline{v}^{\times}\underline{E} \end{cases} \qquad T\underline{v} = -\underline{v}$$

$$\alpha(\underline{v}) = \frac{\varepsilon-\mu}{c}\underline{v}^{\times} \qquad \boxed{\alpha(\underline{v}) = -\tilde{\beta}(-\underline{v})}$$

$$\beta(\underline{v}) = -\frac{\varepsilon-\mu}{c}\underline{v}^{\times}$$

The Theorem of Macroscopical Coefficients Symmetry in Electrodynamics (A. Serdyukov, 1976)



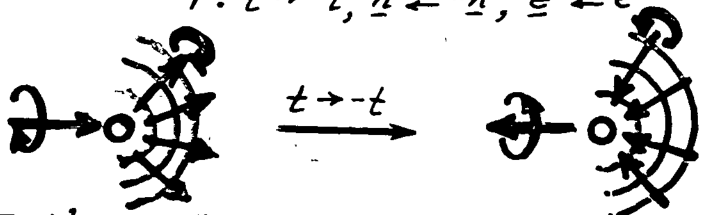
$$\underline{E}' = \frac{1}{c^2 R}\left([[\ddot{\underline{d}}\,\underline{n}']\,\underline{n}'] - [\ddot{\underline{m}}\,\underline{n}']\right)$$

$$\left|\begin{array}{l} \underline{d} = \varkappa \underline{E} + a \underline{H} \\ \underline{m} = b \underline{E} + \chi \underline{H} \end{array}\right|$$

$$\underline{E}' = \underline{e}' \;\frac{\underline{e}^* \underline{E}}{R}\; F(\underline{e}', \underline{n}'; \underline{e}, \underline{n})$$

$$F = \frac{\omega^2}{c^2}\left\{\underline{e}'^* \varkappa \underline{e} - \underline{n}'\underline{n} \cdot \underline{e} \chi \underline{e}'^* + \ldots + \underline{e}'^* a [\underline{n}\,\underline{e}] + [\underline{n}'\underline{e}'^*] b\, \underline{e}\right\}$$

$$T:\; t \to -t,\; \underline{n} \rightleftarrows -\underline{n}',\; \underline{e} \rightleftarrows \underline{e}'^*$$



$$F^T = \frac{\omega^2}{c^2}\left\{\underline{e} \varkappa \underline{e}'^* - \underline{n}\,\underline{n}' \cdot \underline{e}'^* \chi \underline{e} + \ldots - \underline{e}\, a [\underline{n}' \underline{e}'^*] - [\underline{n}\,\underline{e}] b\, \underline{e}'^*\right\}$$

$$\underline{\underline{F^T = F}} \;\longrightarrow\; \boxed{\varkappa = \tilde{\varkappa} \qquad \chi = \tilde{\chi} \qquad a = -\tilde{b}}$$

$$\begin{array}{l} \underline{D} = \underline{E} + 4\pi N \underline{d} \\ \underline{B} = \underline{H} + 4\pi N \underline{m} \end{array} \qquad \left|\begin{array}{l} \underline{D} = \varepsilon \underline{E} + \alpha \underline{H} \\ \underline{B} = \beta \underline{E} + \mu \underline{H} \end{array}\right| \qquad \begin{array}{ll} \varepsilon = 1 + 4\pi N \varkappa & \alpha = 4\pi N a \\ \mu = 1 + 4\pi N \chi & \beta = 4\pi N b \end{array}$$

$$\boxed{\varepsilon = \tilde{\varepsilon} \qquad \mu = \tilde{\mu} \qquad \alpha = -\tilde{\beta}}$$

$$\boxed{\begin{array}{l} \varepsilon(\underline{e}, \underline{h}) = \tilde{\varepsilon}(\underline{e}, -\underline{h}) \\ \mu(\underline{e}, \underline{h}) = \tilde{\mu}(\underline{e}, -\underline{h}) \\ \alpha(\underline{e}, \underline{h}) = -\tilde{\beta}(\underline{e}, -\underline{h}) \end{array}}$$

Duality Transformation and Cinetical Symmetry (1987)

Fenomenology

$$\left\{\begin{pmatrix} \underline{D} \\ \underline{B} \end{pmatrix} = \begin{pmatrix} \varepsilon & \alpha \\ \beta & \mu \end{pmatrix} \begin{pmatrix} \underline{E} \\ \underline{H} \end{pmatrix} \qquad \begin{pmatrix} \underline{D}' \\ \underline{B}' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \underline{D} \\ \underline{B} \end{pmatrix}, \quad \begin{pmatrix} \underline{E}' \\ \underline{H}' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \underline{E} \\ \underline{H} \end{pmatrix}\right.$$

$$\left\{\begin{pmatrix} \underline{D}' \\ \underline{B}' \end{pmatrix} = \begin{pmatrix} \varepsilon' & \alpha' \\ \beta' & \mu' \end{pmatrix} \begin{pmatrix} \underline{E}' \\ \underline{H}' \end{pmatrix} \right. \qquad \begin{pmatrix} \varepsilon' & \alpha' \\ \beta' & \mu' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \varepsilon & \alpha \\ \beta & \mu \end{pmatrix} \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$\begin{aligned} \varepsilon' &= \varepsilon\cos^2\theta + \mu\sin^2\theta - (\alpha+\beta)\cos\theta\sin\theta \\ \mu' &= \mu\cos^2\theta + \varepsilon\sin^2\theta + (\alpha+\beta)\cos\theta\sin\theta \\ \alpha' &= \alpha\cos^2\theta - \beta\sin^2\theta + (\varepsilon-\mu)\cos\theta\sin\theta \\ \beta' &= \beta\cos^2\theta - \alpha\sin^2\theta + (\varepsilon-\mu)\cos\theta\sin\theta \end{aligned} \qquad \left\{\begin{aligned} \underline{D}' &= \varepsilon'(\theta)\underline{E}' + \alpha'(\theta)\underline{H}' \\ \underline{B}' &= \beta'(\theta)\underline{E}' + \mu'(\theta)\underline{H}' \end{aligned}\right.$$

$$T:\quad t \to -t, \quad \underline{E} \to \underline{E}, \quad \underline{H} \to -\underline{H}$$

$$T:\quad \left.\begin{aligned} \underline{E}' &= \underline{E}\cos\theta - \underline{H}\sin\theta \to \underline{E}' \\ \underline{H}' &= \underline{H}\cos\theta + \underline{E}\sin\theta \to -\underline{H}' \end{aligned}\right\} \qquad \underline{\theta \to -\theta}$$

$$\left.\begin{aligned} \varepsilon &= \tilde{\varepsilon} \\ \mu &= \tilde{\mu} \\ \alpha &= -\tilde{\beta} \end{aligned}\right\} \Rightarrow \boxed{\begin{aligned} \varepsilon'(\theta) &= \tilde{\varepsilon}(-\theta) \\ \mu'(\theta) &= \tilde{\mu}(-\theta) \\ \alpha'(\theta) &= -\tilde{\beta}'(-\theta) \end{aligned}}$$

$$\left.\begin{aligned} \underline{D} &= \varepsilon\underline{E} + \alpha H \\ \underline{B} &= \alpha\underline{E} + \mu H \end{aligned}\right\} \quad tg\,2\theta = \frac{2\alpha}{\mu-\varepsilon} \qquad \left.\begin{aligned} \underline{D}' &= \varepsilon'\underline{E}' \\ \underline{B}' &= \mu'\underline{H}' \end{aligned}\right\} \quad \alpha' = 0$$

$$\underline{n = \sqrt{\varepsilon\mu - \alpha^2} = \sqrt{\varepsilon'\mu'}}$$

Particular waves in bi-isotropic media

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PARTICULAR WAVES IN BI-ISOTROPIC MEDIA

I. V. SEMCHENKO

Investigation of the possibility of existence and features of propagation of particular waves (longitudinal and helicoidal) in the magneto-active plasma and metals in the magnetic field are reported in [1,2]. It is demonstrated [3] that similar waves can also exist in natural gyrotropic media within certain frequency ranges close to the absorption band. The present paper results from the study of propagation and excitation of the particular waves in bi-isotropic media having optical properties describable with the help of phenomenological material equations [4-7].

$$\vec{D}=\varepsilon\vec{E}+(x+i\alpha)\vec{H}$$
$$\vec{B}=\mu\vec{H}+(\tilde{x}-i\tilde{\alpha})\vec{E} \tag{1}$$

Here ε, μ and α are tensors of permittivity, magnetic permeability and natural optical activity, x is the tensor describing the medium optical non-reciprocal, the sign "tilde" ($\sim$) designates transposition. Since properties of a medium in material equations (1) are characterized by the tensors ε, μ, α, x they are applicable to describe the optical behaviour of anisotropic optically active non-mutual media.

From Maxwell's equations for flat waves [4]

$$\vec{m}^{x}\vec{E}=\vec{B}, \qquad \vec{m}^{x}\vec{H}=-\vec{D},$$
$$\vec{m}\ \vec{B}=0, \qquad \vec{m}\ \vec{D}=0 \tag{2}$$

and material equations (1) the wave equation for the electric field in the bi-isotropic medium follows as

$$\{\vec{m}\cdot\vec{m}-\vec{m}^{2}+2i\alpha\vec{m}^{x}+\varepsilon\mu-x^{2}-\alpha^{2}\}\ \vec{E}=0$$

and its solution are right-handed and left-handed circular polarized waves with the refraction index

$$n_{\pm}=\sqrt{\varepsilon\mu-x^{2}}\pm\alpha \tag{3}$$

Here $\vec{m}=n\vec{n}$ is the refraction vector, $\vec{n}$ is a unit vector specifying the direction of propagation of waves, $\vec{m}^{x}$ is an antisymmetric tensor dual to the vector $\vec{m}$, the point between the vectors indicates their direct (double) product.

The permittivity within field frequencies close to the medium absorption bands may approach zero making it possible for particular waves in bi-isotropic media to exist. When the relationship

$$\varepsilon\mu = x^2, \tag{4}$$
is fulfilled, which is true for a certain electromagnetic field frequency ω_1 due to the frequency dispersion of the medium parameters, it follows from formula (3) that $n_+ = \alpha$, $n_- = -\alpha$. In this case only a single circularly polarized wave with the refraction index $n = |\alpha|$ can propagate in the bi-isotropic medium:

$$\vec{E}_{\pm} = \mathscr{E}\,\frac{\vec{a} \pm i\vec{b}}{\sqrt{2}} \exp[i(\frac{\omega_1}{c}|\alpha|\vec{n}\,\vec{r} - \omega_1 t)] \tag{5}$$

Here $\vec{a}$ and $\vec{b}$ are single vectors forming the right trio with the vector of the wave normal $\vec{n}$ and the circular polarization sign corresponds to the parametric sign α at the frequency ω_1. From equations (1) and (2) the magnetic field intensity vector $\vec{H}$ is expressed as

$$\vec{H} = \frac{1}{\mu}\{\vec{m} \times \vec{E} - (x - i\alpha)\vec{E}\}$$

Using it and relationship (4) for wave (5) it is obtained

$$\vec{H}_{\pm} = -\frac{1}{\mu}\,x\vec{E}_{\pm} = -\sqrt{\frac{\varepsilon}{\mu}}\,\vec{E}_{\pm}\,,$$
$$\vec{B}_{\pm} = -i\alpha\vec{E}_{\pm}\,,$$
$$\vec{D}_{\pm} = -i\alpha\sqrt{\frac{\varepsilon}{\mu}}\,\vec{E}_{\pm}\,.$$

So, for the above wave it is obtained that $\vec{H} \neq 0$, still Umov-Pointing's vector transforms into zero and the wave does not convey energy. Identical, the so-called spiral, waves can appear also in the magneto-active plasma [1], yet they can propagate only in the direction of an external magnetic field. Now consider the condition

$$\sqrt{\varepsilon\mu - x^2} = \alpha\,, \tag{6}$$
which can occur within a certain frequency ω_2 of the electromagnetic field close to the medium absorption band. When relationship (6) is fulfilled refraction indexes (3) acquire the form

$$n_+(\omega_2) = 2|\alpha(\omega_2)|\,, \tag{7}$$
$$n_-(\omega_2) = 0\,. \tag{8}$$

A circularly polarized electromagnetic wave corresponds to refraction index (7), it is characterized by the following vectors

$$\vec{E} = \mathscr{E}\,\frac{\vec{a} \pm i\vec{b}}{\sqrt{2}}\exp[i(\frac{2\,\omega_2}{c}\,|\,\alpha\,|\,\vec{n}\;\vec{r} - \omega_2 t)]$$

$$\vec{H} = -\frac{1}{\mu}(x + i\alpha)\vec{E}\,,$$

$$\vec{D} = -\frac{2i\alpha}{\mu}(x + i\alpha)\;\vec{E}\,,$$

$$\vec{B} = -2i\alpha\,\vec{E}$$

where the circular polarization sign is determined by the optical activity parameter sign $\alpha(\omega_2)$.
A wave with arbitrary elipticity γ corresponds to the zero refraction index

$$\vec{E} = \mathscr{E}\,\frac{\vec{a} \pm i\gamma\vec{b}}{\sqrt{1 + \gamma^2}}\,e^{-i\omega_2 t}$$

$$\vec{H} = \frac{1}{\mu}(i\alpha - x)\vec{E}\,,$$

$$\vec{B} = \vec{D} = 0\;.$$

which has an infinite phase velocity and is independent of coordinates. The energy flux density vector of the wave averaged in time has the form

$$<\vec{S}>_t = -\frac{c}{4\pi}\frac{\alpha\gamma\mathscr{E}^2}{\mu\,(1 + \gamma^2)}\vec{n}$$

Longitudinal waves of the plasma type can also exist within the frequency ω_2 for which condition (6) is fulfilled:

$$\vec{E} = \mathscr{E}\vec{n}\exp[i(K_{||}\vec{n}\,\vec{r} - \omega_2 t)]$$

$$\vec{B} = \vec{D} = 0\;. \tag{9}$$

$$\vec{H} = -\frac{1}{\mu}(x - i\alpha)\vec{E},$$

The wave number $K_{||}$ of the longitudinal waves can be determined by considering the second-order three-dimensional dispersion taking into account that the permittivity tensor depends upon the wave number [1]:

$$\varepsilon(\omega, \vec{k}) = \varepsilon_0(\omega) + \gamma_0(\omega)\vec{k}^2 + (\gamma_1(\omega) - \gamma_0(\omega))\vec{k}\,\vec{k} \tag{10}$$

Then the wave equation of the electrical field intensity has the following form:

$$\{\varepsilon_0\mu - x^2 - \alpha^2 - (1 - \beta_0\mu)\vec{m}^2 + [1 + (\beta_1 - \beta_0)\mu]\vec{m}\cdot\vec{m} + 2i\alpha\vec{m}^{x}\}\vec{E} = 0\ , \tag{11}$$

where the following designations are used:

$$\beta_0 = \gamma_0\frac{\omega^2}{c^2}, \qquad \beta_1 = \gamma_1\frac{\omega^2}{c^2}\ .$$

From wave equation (11) a condition of the electrical field longitudinal component follows

$$(\varepsilon_0\mu - x^2 - \alpha^2 + \beta_1\mu n^2)E_{||} = 0\ ,$$

from which the longitudinal wave number is derived

$$K_{||} = \frac{\omega}{c}\sqrt{\frac{\alpha^2 + x^2 - \varepsilon_0\mu}{\beta_1\mu}}\ . \tag{12}$$

In response to the parameter sign β_1 the non-attenuating longitudinal waves can exist either within the frequency range in which

$$\alpha^2 + x^2 > \varepsilon_0\mu \qquad (\text{at } \beta_1 > 0)$$

or within the frequency range in which

$$\alpha^2 + x^2 < \varepsilon_0\mu \qquad (\text{at } \beta_1 < 0)$$

Since the energy flux density vector of waves with refraction indexes (7) and (8) differs from zero these waves can be excited by an electromagnetic wave incident upon a bi-isotropic medium from outside. One of mechanisms exciting a longitudinal wave (9) can be the effect of Vavilov-Cherenkov which is discussed below. For this purpose the wave equation of the electromagnetic field with the source in the bi-isotropic medium is written as:

$$\{rot\,rot + \frac{\varepsilon\mu - x^2 - \alpha^2}{c^2}\frac{\partial^2}{\partial t^2} - \frac{2i\alpha}{c}\frac{\partial}{\partial t}\,rot\}\vec{E} = -\frac{4\pi\mu}{c^2}\frac{\partial}{\partial t}\vec{j}\ .$$

When second-order three dimensional dispersion (10) is taken into account the wave equation acquires the form

$$\{[1 - \frac{\gamma_1 - \gamma_0}{c^2}\mu\frac{\partial^2}{\partial t^2}]\nabla\cdot\nabla - (1 + \frac{\gamma_0}{c^2}\mu\frac{\partial^2}{\partial t^2})\nabla^2 + \tag{13}$$

$$+\frac{\varepsilon_0\mu - x^2 - \alpha^2}{c^2}\frac{\partial^2}{\partial t^2} - \frac{2i\alpha}{c}\frac{\partial}{\partial t}\nabla^x\}\vec{E} = -\frac{4\pi\mu}{c^2}\frac{\partial}{\partial t}\vec{j}\ .$$

Where ∇ is a symbolic vector differential operator. In order to take into account the frequency dispersion of the medium properties the parameters ε_0, μ, x, $\alpha, \gamma_0\ \gamma_1$ should be treated as differential argument $i\frac{\partial}{\partial t}$ - dependent operators. By writing down the current density for a pin-pointed charge e moving with a constant velocity $\vec{v}$

$$\vec{j} = e\vec{v}\delta(\vec{r} - \vec{v}t)\ ,$$

non-homogeneous equation (13) is solved for the Fourier-component of the longitudinal field:

$$\vec{E}_1 = -\frac{4\pi\mu e i\vec{k}}{k^2(\varepsilon_0\mu - x^2 - \alpha^2 + \gamma_1\mu k^2)}e^{i\vec{k}(\vec{r} - \vec{v}t)}\ .$$

Also,the solution of equation (13) with the zero right term should be taken into account

$$\vec{E}_2 = E_{20}\vec{k}e^{i(\vec{k}\vec{r} - \omega t)}\ .$$

Now a full solution of equation (13)

$$\vec{E} = \vec{E}_1 + \vec{E}_2\ ,$$

satisfying the original condition

$$E(\vec{r}, t) = 0 \ ,$$

at t=0 acquires the form

$$\vec{E}(\vec{r},t) = \frac{4\pi\mu e i\vec{k}}{k^2} \frac{1 - e^{-i(\vec{k}\vec{v} - \omega)t}}{\varepsilon_0\mu - x^2 - \alpha^2 + \gamma_1\mu k^2} e^{i(\vec{k}\vec{r} - \omega t)} \ , \tag{14}$$

where all the parameters characterizing a medium are functions of the argument $\vec{k}\vec{v}$.

When the conditions

$$\varepsilon_0\mu - x^2 - \alpha^2 + \gamma_1\mu k^2 = 0 \ , \qquad \vec{k}\vec{v} = \omega \tag{15}$$

are fulfilled field amplitude (14) increases in time linearly:

$$\vec{E}(\vec{r},t) = -\frac{4\pi\mu e i\vec{k}t}{k^2\mu\dfrac{\partial\varepsilon_0(\omega)}{\partial\omega}} e^{i(\vec{k}\vec{r} - \omega t)} \ ,$$

It has been assumed that magnitudes x^2, α^2 and $\gamma_1 k^2$ are less frequency-dependent than ε_0.

From relationships (12), (15) it follows that the Cherenkov emission of longitudinal waves is possible providing

$$v \geq \omega \sqrt{\frac{\gamma_1\mu}{\alpha^2 + x^2 - \varepsilon_0\mu}} \ ,$$

i.e., when the velocity of an electron exceeds the phase velocity of longitudinal waves in a bi-isotropic medium.

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